# **DIY: Solving Quadratic Equations**

To review the methods of solving quadratic equations, click on the following links to watch the following YouTube videos. The videos go over various methods of solving quadratic equations including factoring, square root property, completing the square and quadratic formula. They are followed by several practice problems for you to try, covering all the basic concepts covered in the video, with answers and detailed solutions. Some additional resources are included for more practice at the end.

- 1. <u>https://www.youtube.com/watch?v=8QHpFfnqJJg</u> Introduction to quadratic equations.
- 2. Solving Quadratic Equations by Factoring
  - a. https://www.youtube.com/watch?v=SDe-1lGeS0U Guess and Check method
  - b. <u>https://www.youtube.com/watch?v=HvBiJ9W00Z4</u> Factoring by grouping. (The presenter says he is factoring "trinomials", but he is only using quadratic expressions. A trinomial can be any 3-term polynomial expression, though if it is not of a quadratic form, the method taught here cannot be used.)
- 3. <u>https://www.youtube.com/watch?v=Fj-BP7uaWrI</u> Solving quadratic equations by the square root property.
- 4. Solving Quadratic Equations by Completing the square
  - a. <u>https://www.youtube.com/watch?v=sv0L9JpkCv0</u> Using the method with leading coefficient = 1
  - b. <u>https://www.youtube.com/watch?v= dlbtdsT198</u> Using the method with leading coefficient ≠ 1
- 5. Solving Quadratic Equations by Quadratic Formula for equations with:
  - a. <u>https://www.youtube.com/watch?v=JSwjmTFMDwg</u> two rational roots
  - b. <u>https://www.youtube.com/watch?v=H5AM1bzqCQw</u> two irrational, real roots
  - c. <u>https://www.youtube.com/watch?v=rziJBuMI5Ts</u> two complex roots
- 6. <u>https://www.youtube.com/watch?v=px4MUZVi4Qw</u> Predicting the type of solutions using the discriminant
- 7. Application of quadratic equations
  - a. <u>https://www.youtube.com/watch?v=IGGnn9oa4QY</u>
  - b. <u>https://www.youtube.com/watch?v=-q0y2YBIX-U</u>
  - c. <u>https://www.youtube.com/watch?v=RdfWo0ae53o</u>
  - d. <u>https://www.youtube.com/watch?v=TzRQSD3iQA4</u>



**Practice problems**: The following problems use the techniques demonstrated in the above video. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact <u>Math Assistance Area</u>.

- 1. State if the following equations are quadratic equations or not and why? a) 2x+3y=5 b)  $8x^2 + 22 - 5x = 10$  c)  $10x^{12} - 15x^2 = 10$  d)  $7x^2 + 4x = 0$
- 2. Solve the following quadratic equations using factoring: a)  $x^2 + 7x + 15 = 5$  b)  $x^2 - 2x - 15 = 0$  c)  $8x^2 + 40x - 8x + 32 = 0$  d)  $9x^2 - 16 = 0$
- 3. Solve by using guess and check method for factoring: a)  $14x^2 - 4x - 15 = -5$ b)  $110x^2 + 23x - 3 = 0$ c)  $-x^2 - 27x - 96 = x$ d)  $2x^2 + 19x + 35 = 0$
- 4. Solve the following quadratic equations using square-root property a)  $x^2 + 2x - 9 = 2x$  b)  $4x^2 - 31 = 1$  c)  $3x^2 + 27 = -13x^2 + 9$  d)  $x^2 + 30 = 5$
- 5. Solve the following quadratic equations using completing the square a)  $x^2 + 8x - 9 = 0$  b)  $x^2 + 3x + 5 = 100$  c)  $2x^2 + 5x - 12 = 0$  d)  $3x^2 - x + 3 = 0$
- 6. Solve the following quadratic equations using quadratic formula a)  $x^2 + 9x + 20 = 0$  b)  $16x^2 + 24x + 29 = 0$  c)  $12x^2 + 6x - 1 = 0$  d)  $144x^2 + 72x + 4 = 0$ 
  - e)  $20x^2 + 24x 1 = 7x^2 3$  f)  $-x^2 + 2x + 11 = 0$  g)  $-16x^2 + 40x 25 = 0$
- 7. Use the discriminant to predict the number and types of solutions of the following quadratic equations a)  $2x^2 + 8x + 8 = 0$ b)  $3x^2 + 5x + 1 = 0$ c)  $7x^2 - 28 = 0$ d)  $9x^2 + 11x + 8 = 0$
- 8. A picture has dimensions of 12in by 30in. Joe wants to frame the picture with a border such that the total covered area is 648 sq. in. How wide is the border?
- 9. Find the legs and hypotenuse of a right triangle if one of the legs is 14 inches less than the longer leg and the hypotenuse is 2 inches longer than the longer leg. (Hint: use the Pythagorean Theorem. If the lengths of the legs of a right triangle are a and b and the length of the hypotenuse is c, then the relation between these lengths is always  $a^2 + b^2 = c^2$ .)
- 10. The height of a projectile is given by  $h = -16t^2 + v_0t + h_0$  where  $v_0$  is the initial velocity and  $h_0$  is the initial height of the projectile.
  - a. Calculate the time it takes the projectile to reach a height of 60ft when it is projected with a velocity of 120ft/sec from a height of 10ft above the ground.

b.At what time does the projectile touch the ground? (*Note: Round your answer(s) to the nearest hundredth.*)



Answers:

1. c) No a) No b) Yes d) Yes 2. a) {-2,-5} b) {-3,5} c) {-2} d)  $\{\frac{4}{3}, -\frac{4}{3}\}$ 3. a)  $\{-\frac{5}{7}, 1\}$  b)  $\{\frac{1}{11}, -\frac{3}{10}\}$  c)  $\{-24, -4\}$  d)  $\{-\frac{5}{2}, -7\}$ 4. 4. a) {3,-3} b) { $2\sqrt{2}$ , -  $2\sqrt{2}$ c) { $\frac{3\sqrt{2}}{4}i$ , - $\frac{3\sqrt{2}}{4}i$ } d) {5i, -5i} 5. a)  $\{1,-9\}$  b)  $\{\frac{-3+\sqrt{389}}{2}, \frac{-3-\sqrt{389}}{2}\}$  c)  $\{-4, \frac{3}{2}\}$  d)  $\{\frac{1+i\sqrt{35}}{6}, \frac{1-i\sqrt{35}}{6}\}$ 6. a) {-4,-5} b)  $\{\frac{-3 \pm 2 i\sqrt{5}}{4}\}$  c)  $\{\frac{-3 \pm \sqrt{21}}{12}\}$  d)  $\{\frac{-3 \pm \sqrt{5}}{12}\}$ e)  $\{\frac{-12 \pm \sqrt{118}}{13}\}$  f)  $\{1 \pm 2\sqrt{3}\}$  g)  $\{\frac{5}{4}\}$ 7. a) One Real b) Two real, irrational c) Two real, rational d) Two Complex Solution Solutions solutions Solutions

8. The width of the picture frame is 3 in.

9. The lengths of the legs are 10 and 24 in. and the hypotenuse is 26 in.

10.

a) t=7.06 sec and t=0.44 sec

b) t=7.58 sec



$$\begin{array}{c} \hline Detailed Solution\\ \hline$$

$$\begin{array}{c} 3a) \ 14x^{2} - 4x - 16 = -5 \\ +5 + 5 \\ \hline 14x^{2} - 4x - 10 = 0 \\ (7x + 5) \ (2x - 2) = 0 \\ \text{Hence} \\ 7x + 5 \ (2x - 2) = 0 \\ \text{Hence} \\ 7x + 5 \ (2x - 2) = 0 \\ \text{Hence} \\ 7x + 5 \ (2x - 2) = 0 \\ \hline 11x - 1 \ = 0 \ ov \ 10x + 3 = 0 \\ \text{Hence} \\ 11x - 1 \ = 0 \ ov \ 10x + 3 = 0 \\ \text{Hence} \\ 11x - 1 \ = 0 \ ov \ 10x + 3 = 0 \\ \text{Hence} \\ 11x = 1 \\ \hline 11 \ 11 \ 11 \ 120 \ 10x = -3 \\ \hline 11x = 1 \\ \hline 1x = 1 \\ \hline 1x = -3 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ ov \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \ v \ x + 7 = 0 \\ \hline 2x + 6 = 0 \\$$







6.c) 
$$|3x^{2} + 4x - 1| = 0$$
  
 $a \ge 12$   $b \ge 6$   $c \ge -1$   
 $x = -6 \pm \sqrt{6^{2} - 4(12)(-1)}$   
 $= -6 \pm \sqrt{36 + 48} = -6 \pm \sqrt{84}$   
 $24$   
 $x = -6 \pm \sqrt{36 + 48} = -6 \pm \sqrt{84}$   
 $24$   
 $x = -6 \pm \sqrt{84}$   
 $x = -6 \pm \sqrt{84}$   

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(6d)  $144x^{2} + 72x + 4 = 0$  (all coefficients can be divided by 4).  $36x^{2} + 18x + 1 = 0$  a=36 b=18 d=1  $x = -\frac{18 \pm \sqrt{18^{2} + 4(3c)(1)}}{2(3c)} = -\frac{18 \pm \sqrt{324 - 144}}{72} = -\frac{18 \pm \sqrt{180}}{72}$   $= -\frac{18 \pm \sqrt{9.4.5}}{72} = -\frac{18 \pm 3.2\sqrt{5}}{72} = -\frac{18 \pm 6\sqrt{5}}{72}$  $x = \frac{16(-3 \pm \sqrt{5})}{\frac{12}{72}}$   $x = -\frac{3 \pm \sqrt{5}}{12}$ 

6e) 
$$20x^{2} + 24x - 1 = 7x^{2} - 3$$
  
 $\frac{-7x^{2}}{13x^{2} + 24y + 2} = 0$   $a = 13$   $b = 24$   $c = 2$   
 $x = -\frac{24 \pm \sqrt{24^{2} + 4(13)(2)}}{2(13)} = -\frac{24 \pm \sqrt{472}}{24} = -\frac{24 \pm 2\sqrt{118}}{24}$   
 $= \frac{2(-12 \pm \sqrt{118})}{2(13)}$   $x = -\frac{12 \pm \sqrt{118}}{13}$ 

$$(4^{2}) - x^{2} + 2x + 11 = 0 \qquad a = -1 \qquad b = 2 \qquad c = 11$$

$$x = -\frac{2 \pm \sqrt{2^{2} + (4)(5)}}{2(-1)} = -\frac{2 \pm \sqrt{44 + 444}}{-2}$$

$$= -\frac{2 \pm \sqrt{45}}{-2} = -\frac{2 \pm \sqrt{43}}{-2} = -\frac{2 \pm \sqrt{43}}{-2} = -\frac{4}{-2} \left(\frac{1 \pm 2\sqrt{3}}{2}\right)^{4}$$

$$x = \frac{1 \pm 2\sqrt{3}}{-2} \qquad \text{when fadowing out the}$$

$$(-2), \text{ since we are using both + 4(5, our resolvent) will be - 2\sqrt{5}, 2\sqrt{3}$$

$$(-2), \text{ the same Mooth as (-4), -2 = 0 \qquad (modth) + \sqrt{2} + 40x + 25 = 0$$

$$x = -\frac{40 \pm \sqrt{40^{2} + 4(-1)(-25)}}{2(-4)} = -\frac{40 \pm \sqrt{1600 - 1600}}{-32}$$

$$x = -\frac{40 \pm \sqrt{40^{2} - 4(-1)(-25)}}{-32} = -\frac{40 \pm \sqrt{1600 - 1600}}{-32}$$

$$(-2), -2) = -\frac{40 \pm \sqrt{1600}}{-32} = -\frac{40}{-32} = -\frac{40}{4} = \frac{5}{4}$$

$$(-2), -2) = -\frac{40}{-32} = -\frac{40}{-32} = -\frac{40}{4} = \frac{5}{4}$$

-6-



(7b)  $3x^2 + 5x + 1 = 0$ 7 a) 2x2+8x+8=0 Discriminant Discoundant  $D = b^2 - 4ac$  $D = b^2 - 4ac$ a=3, b=5, c=1 a=2, b=8, C=8  $D = 5^2 - 4(3)(1)$ = 25 - 12 = 13 Therefore  $D = 8^2 - 4(2)(8)$ since D>0 and not a perfect square we have = 64-64 two irrational solutions D = 0 There is one real solution 7d) 9x2+11x +8 =0 7c)  $7\pi^2 - 28 = 0$  $D = b^2 - 4ac$ Notice the equation is missing a=9, b=11, c=8 the x ferm ! Hence  $\mathcal{D} = 11^2 - 4(9)(8)$ 7n² + 0x - 28 = 0 = 121 - 28.8 Now D= b-4ac =-167 therefore we since D < O therefore we a=7, b=0, c=-28  $D = 0^2 - 4(7)(-28)$ have two complex = (28)(-28)=(28)<sup>2</sup> solutions. = 784 Now D>0 and is a perfect square hence we have two rational solution Let the width of the frame be x inches. hence the total area of the picture 8) is given by (30+2x)(12+2x) = 648(30 FIX) 360 +60x +24x +4x2 = 648  $4x^2 + 84x + 360 = 648$  $x^{2} + 21 \times +90 = 162$ -162 - 162 /12+2x)->  $\chi^2 + 21 - 72 = 0$ (x-3)(x+24) = 0

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## **Additional Resources**

Click on the links below to download worksheets under "Basics" for more practice:

- 1. <u>Completing the square</u>
- 2. Solving quadratic equations
  - <u>by taking square roots</u>
  - <u>by factoring</u>
  - <u>with the quadratic formula</u>
  - <u>by completing the square</u>

### Alternatively;

- 1. Go To <a href="http://www.kutasoftware.com/free.html">http://www.kutasoftware.com/free.html</a>
- 2. Under "Quadratic Functions" click on:
  - 3. Completing the square
  - 4. Solving quadratic equations
    - by taking square roots
    - by factoring
    - with the quadratic formula
    - by completing the square

You can print out the worksheets and work on them. The solutions are provided at the end of the worksheets

3. For help please contact the *Math Assistance Area*.

