DIY: *Graphing Functions: Polynomials*

To review graphs of polynomial functions, watch the following set of YouTube videos explaining what polynomial functions are, how to sketch their graphs, and how to determine the function from the graph.

They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=vpTvNFXvj38 Graphing quadratic functions
2. https://www.youtube.com/watch?v=GALfCd-2XRQ&list=PL4FB17E5C77DCCE69&index=2 Identifying characteristics of graphs (increasing, decreasing, constant, maxima/minima)
3. https://www.youtube.com/watch?v=of2OG5NNNIo graphing polynomials, part 1
4. https://www.youtube.com/watch?v=qi9ZlTiHIwY graphing polynomials, part 2
5. https://www.youtube.com/watch?v=Ur1AWcKIIvM finding a polynomial function from its graph, part 1
6. https://www.youtube.com/watch?v=lz-gNP_D7gE finding a polynomial function from its graph, part 2
7. https://www.youtube.com/watch?v=EEB13cADwFg finding the remaining zeros of a polynomial when some of the zeros and their multiplicities are given. (Note: this presenter has other videos on this topic which may be helpful. They will automatically follow this video.)

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Sketch the graph of $f(x) = -\frac{1}{2}(x+3)^2 + 4$. Include the coordinates of the vertex.

2. Sketch the graph of $g(x) = 2x^2 + x - 6$, including the coordinates of the vertex.

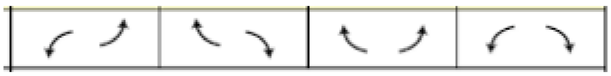
3. Complete the table with a small sketch of which direction the ends of each type of polynomial function would take.

Sign of Leading Coefficient	Odd Degree	Even Degree
+		
-		

4. Given the function $f(x) = (x-2)(x+1)^2(2x-5)^4$:

   a. What is the degree of $f(x)$?

   b. Which sketch would best approximate how the ends of the graph of $f(x)$ appear?



   c. For what x-values would f(x) = 0? (These are the x-intercepts)

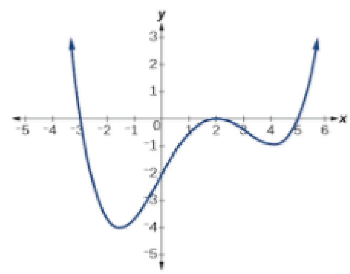
   d. What are the coordinates of the y-intercept.

   e. For which of the zeros (x-intercepts) would the sign of f(x) change? (Hint: think what would happen on the graph when the sign of f(x) changes).

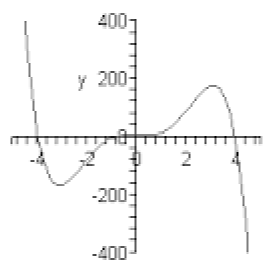
5. Sketch the graph of $y = -(x+2)^3(x-1)(x-3)^2$

6. Sketch the graph of $y = x^3 - 4x^2$.

7. Find the equation for $f(x)$ given that its graph is shown here.



8. a. Find $f(x)$ for the function whose graph is



   b. If $f(x)$ an odd function, even function or neither?

   c. What degree polynomial is $f(x)$?

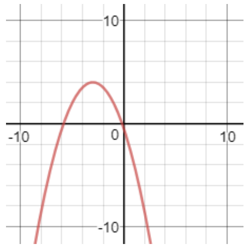
9. We are told that the function $f(x) = x^4 + 6x^3 - 16x^2 - 150x - 225$ has a zero at x = -3 with a multiplicity of 2. Find all the zeros of $f(x)$. (Hint: in the factored form, what would have to appear?)

10. An object if thrown upward with a velocity of 16 ft/sec/sec from a height of 10 feet above the ground. Its height $h(t)$ at any time t sec. is given by $h(t) = -16t^2 + 16t + 10$. How many seconds after it was thrown will the object reach its maximum height and when will it hit the ground?

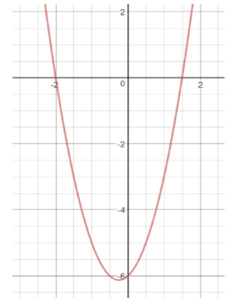
(see the next page for the answers to these problems)

Answers to Practice Problems:

1. Vertex: (-3, 4)



2. Vertex: $\left(-\frac{1}{4}, -6\frac{1}{8}\right)$



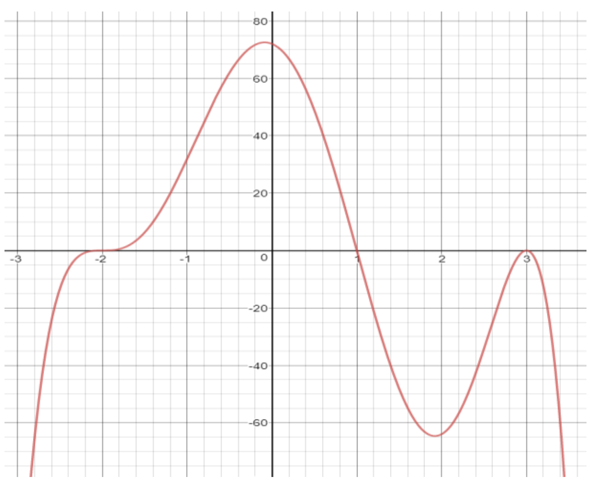
3. (see completed table in the Detailed Solutions section)

4. a. degree = 7

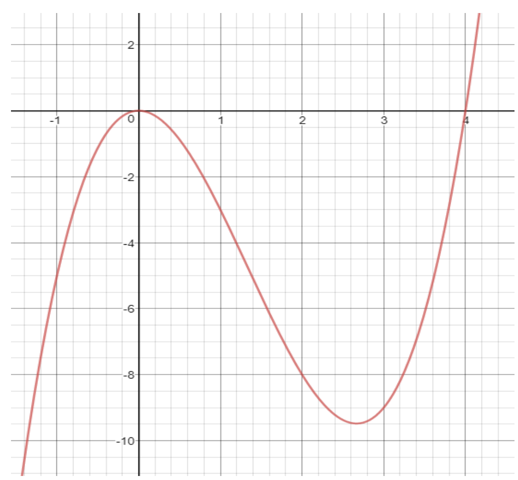
b.

c. { 2, -1, 5/2 } d. (0, -1250) e. x = 2 only

5.



6.



7. $f(x) = \frac{1}{30}(x+3)(x-2)^2(x-5)$

8. a. $f(x) = -\frac{5}{12}x^3(x+4)(x-4)$ *or*

$$f(x) = -\frac{5}{12}x^5 + \frac{20}{3}x^3$$

b. f(x) is an odd function

c. degree = 5

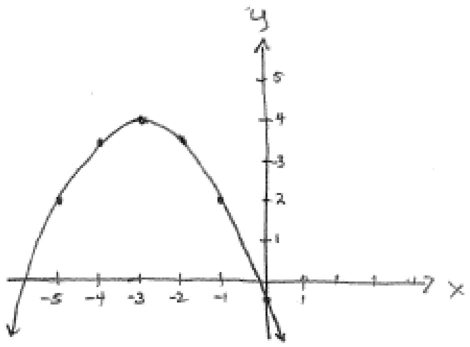
9. {-3, 5, -5 } (Note: since -3 is a zero of multiplicity 2, this is actually 4 zeros.)

10. a. $\frac{1}{2}$ sec. b. $\frac{2\pm\sqrt{14}}{4} \approx$ 1.435 sec.

(See the next page for the detailed solutions.)

Detailed Solutions:

1. $f(x) = -\frac{1}{2}(x+3)^2 + 4$



The x-coordinate of the vertex, when the function is written in the form $y = a(x-h)^2 + k$, is the value of x that makes $(x-h) = 0$ (namely, $x=h$).

Here, $x+3=0$ when $x=-3$

note: $(x+3) = (x-\underbrace{(-3)}_{h})$

The y-coordinate of the vertex is k. since, if $x=h$ $\quad y = a(h-h)^2+k$
$= 0+k$
$y=k$.

Here, when $x=-3$, $y=4$

vertex: $(-3,4)$. To draw the rest of the graph, make a table of values with ← some points on either side of the vertex.

	x	$f(x)=y$
	-5	2
	-4	$3\frac{1}{2}$
vertex:	-3	4
	-2	$3\frac{1}{2}$
	-1	2
	0	$-\frac{1}{2}$
	1	-4

← for example: $f(-2) = -\frac{1}{2}(-2+3)^2+4 = -\frac{1}{2}(1)^2+4 = \frac{-1}{2}+4 = 3\frac{1}{2}$

2. $g(x) = 2x^2 + x - 6$

This function is not in vertex form, so use the formula $h = -\frac{b}{2a}$. For $g(x)$, $a=2$ $b=1$ $c=-6$

$h = \frac{-1}{2(2)} = -\frac{1}{4}$

To find k, $g(-\frac{1}{4}) = 2(-\frac{1}{4})^2 + (-\frac{1}{4}) - 6$

$= 2(\frac{1}{16}) - \frac{1}{4} - 6 \cdot \frac{4}{4}$

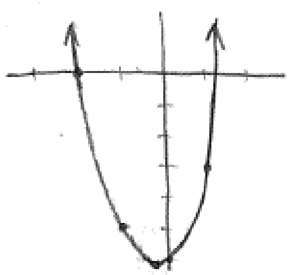
$= \frac{1}{8} - \frac{1}{4} - \frac{24}{4} = \frac{1}{8} - \frac{25}{4} \cdot \frac{2}{2}$

$= \frac{1}{8} - \frac{50}{8} = \frac{-49}{8} = -6.125$

vertex: $(-\frac{1}{4}, -\frac{49}{8}) = (-.25, -6.125)$

table of values:

x	$g(x)$
-2	0
-1	-5
$-\frac{1}{4}$	$-49/8 = -6\frac{1}{8}$
0	-6
1	-3



3

Sign of Leading Coefficient	Odd Degree	Even Degree
+	↙ ↗	↖ ↗
-	↖ ↘	↙ ↘

Odd degree: ends go in opposite directions

Even degree: ends go in same direction

(+) leading coefficient: right end goes up.

(−) leading coefficient: right end goes down.

4. $f(x) = (x-2)(x+1)^2(2x-5)^4$

a. $f(x) = (x\ldots)(x^2\ldots\ldots)(16x^4\ldots\ldots)$

$= x(x^2)(16x^4) + \ldots = 16x^7 + \ldots$ $f(x)$ is a 7^{th} degree polynomial.

alternate method: add exponents on all factors as long as all x's inside ( ) are to first power.

$(\quad)^1(\quad)^2(\quad)^4$

degree $= 1+2+4 = 7$

b. Since the leading term of $f(x)$ is an odd degree with a (+) coefficient, the ends will be most like the first pair: ↙ ↗

c. $f(x) = 0$

$(x-2)(x+1)^2(2x-5)^4 = 0$ where $x-2=0 \rightarrow \boxed{x=2}$

or $x+1=0 \rightarrow \boxed{x=-1}$

or $2x-5=0 \rightarrow \boxed{x=5/2}$

d. y-intercept is y-value when $x=0$

$f(0) = (0-2)(0+1)^2(2\cdot 0-5)^4 = (-2)(1)^2(-5)^4$

$= (-2)(1)(625) = -1250$

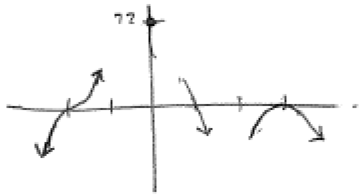
coordinates: $\boxed{(0, -1250)}$

e. Zeros of odd multiplicity (zeros of factors with odd exponents) will have the graph passing through the x-axis, while zeros of even multiplicity "bounce" off the x-axis. Therefore, the function will only change sign (pass through the x-axis) at zeros of odd multiplicity. In this function, $\boxed{\text{y will change sign at } x=2 \text{ only!}}$

5. $y = -(x+2)^3(x-1)(x-3)^2$

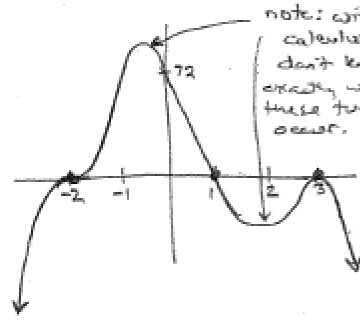
degree $= 3+1+2 = 6$ (even)
leading coefficient $(-)$.
end behavior: ↙ ↘

zeros: -2 (multiplicity 3) shape: (cubic)
1 (multip. 1) shape: (linear)
3 (multip. 2) shape: (parabolic)



y-intercept: $y = -(2)^3(-1)(-3)^2$
$= -8(-1)(9)$
$= +72$

now, connect the pieces with a smooth curve:



note: without calculus, we don't know exactly where these turns occur.

No table of values was made, so this graph is merely a rough sketch

(See the short answer section for #5 to see what the graph looks like exactly, to scale.).

6. $y = x^3 - 4x^2$
$= x^2(x-4)$

degree $= 3$ (odd)
leading coefficient $= +$
} end behavior: ↙ ↗

zeros: $x=0$ (multiplicity 2) shape:
$x=4$ (multip. 1) shape:

preliminary sketch

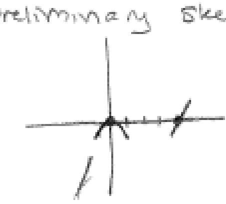
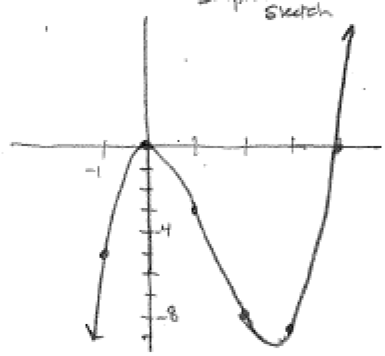


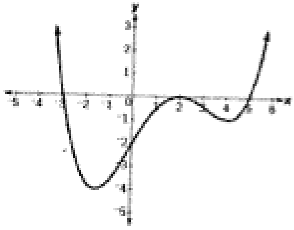
Table of values } find a few points between the zeros and beyond the zeros.

x	y
-1	-5
0	0
1	-3
2	-8
3	-9
4	0
5	25

Improved sketch



7.



with end behavior ↑ ↗, the function must be of even degree with a positive leading coefficient.

Zeros: $x=-3$, multiplicity 1
$x=2$, multip. 2
$x=5$, multip. 1

degree of function $= 1+2+1 = 4$

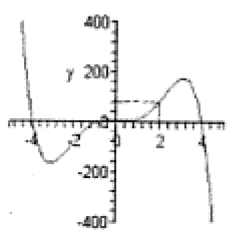
$$f(x) = a(x-(-3))^1(x-2)^2(x-5)^1$$
$$= a(x+3)(x-2)^2(x-5)$$

the correct leading coefficient, a, can be found using the y-intercept: the point $(0,-2)$ is on the graph of $f(x)$

$$f(0) = a(0+3)(0-2)^2(0-5) = -2$$
$$a(3)(4)(-5) = -2$$
$$-60a = -2$$
$$a = \frac{-2}{-60} = \frac{1}{30}$$

$$\boxed{f(x) = \frac{1}{30}(x+3)(x-2)^2(x-5)}$$

8. a.



with end behavior ↑ ↓

ends in opposite directions: odd degree
right end headed down: leading coefficient (−)

Zeros: $x=-4$ (multiplicity 1)
$x=0$ (multiplicity 3)
$x=4$ (multiplicity 1)

degree $= 1+3+1 = 5$

$$f(x) = a(x-(-4))^1(x-0)^3(x-4)^1$$
$$= a(x+4)x^3(x-4)$$ note: a will be negative.

In this case, the y-intercept is 0, so the leading coefficient can't be found using this point. It appears that $(2,40)$ is a point on $f(x)$.

$$f(2) = a(2+4)(2^3)(2-4) = 40$$
$$a(6)(8)(-2) = 40$$
$$-96a = 40 \qquad a = -\frac{40}{96} = \frac{-5}{12}$$

$$f(x) = -\frac{5}{12}(x+4)x^3(x-4) \quad \text{or} \quad -\frac{5}{12}x^3(x^2-16) = \boxed{-\frac{5}{12}x^5 + \frac{20}{3}x^3}$$

8.b. Note the symmetry of the graph of f(x). This is characteristic of a function which is symmetric with respect to the origin, or an odd function. For each point on the graph (a,b), there is a point (−a,−b).

Note also, that each term of the function, in the multiplied-out form, is an odd-degree polynomial term. This is also a way to determine f(x) is an odd function.

Formal proof: $f(-x) = -\frac{5}{12}(-x)^5 + \frac{20}{3}(-x)^3$

$$= -\frac{5}{12}(-x^5) + \frac{20}{3}(-x^3)$$

$$= \frac{5}{12}x^5 - \frac{20}{3}x^3 = -f(x)$$

since $f(-x) = -f(x)$, $f(x)$ is an odd function

c. since the highest power of x is x^5, f(x) is a 5^{th} degree polynomial.

9. $f(x) = x^4 + 6x^3 - 16x^2 - 150x - 225$ has a zero of multiplicity 2 at $x = -3$. So

$$f(x) = (x-(-3))^2 \cdot q(x)$$
$$= (x+3)^2 \cdot q(x)$$

To find q(x), divide.

$$\begin{array}{r|l}
 & x^3 + 3x^2 - 25x - 75 \\
x+3 & x^4 + 6x^3 - 16x^2 - 150x - 225
\end{array} \longrightarrow \frac{x^4}{x} = x^3$$

$x^3(x+3) \to -(x^4+3x^3)$

$3x^3 - 16x^2 \longrightarrow \frac{3x^3}{x} = 3x^2$

$3x^2(x+3) \longrightarrow -(3x^3+9x^2)$

$-25x^2 - 150x \longrightarrow \frac{-25x^2}{x} = -25x$

$-25x(x+3) \longrightarrow -(-25x^2 - 75x)$

$-75x - 225 \longrightarrow \frac{-75x}{x} = -75$

$-75(x+3) \longrightarrow (-75x - 225)$

0 ← we should get 0 as the remainder.

9. (continued) $f(x) = (x+3)(x^3+3x^2-25x-75)$

now divide the second factor $(x^3+3x^2-25x-75)$ again by $x+3$

This time, using synthetic division:

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -25 & -75 \\ & \downarrow & -3 & 0 & 75 \\ \hline & 1 & 0 & -25 & 0 \end{array}$$

$f(x) = (x+3)(x+3)(x^2-25)$

$f(x) = (x+3)^2(x-5)(x+5)$

$\boxed{\text{Zeros of } f(x):\ -3 \text{ (multiplicity 2)},\ 5, \text{ and } -5.}$

10. a. $h(t) = -16t^2+16t+10$. This is a quadratic function with a negative leading coefficient, so its shape is $\cap$ (parabola).
The maximum h will occur at the vertex.

t-coordinate of vertex: $t = \frac{-b}{2a} = \frac{-16}{2(-16)} = \boxed{\frac{1}{2} \text{ sec.}}$

The object will reach its maximum height at $t = 1/2$ sec.

$h(1/2) = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 10 = -16\left(\frac{1}{4}\right) + 8 + 10 = -4 + 18$
$= 14$ feet

b. it will hit the ground when $h = 0$.

$0 = -16t^2 + 16t + 10$

using the quadratic formula, $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-16 \pm \sqrt{16^2-4(-16)(10)}}{2(-16)}$

$= \frac{-16 \pm \sqrt{256+640}}{-32} = \frac{-16 \pm \sqrt{896}}{-32}$ *

$= \frac{-16 \pm 8\sqrt{14}}{-32} = \frac{-16}{-32} \pm \frac{8\sqrt{14}}{-32}$

$= \frac{1}{2} \mp \frac{\sqrt{14}}{4} \approx .5 \pm .935 = -.435$
or $\boxed{1.435 \text{ sec.}}$

* $896 = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot7$
$= 2^7\cdot7$

$\sqrt{896} = \sqrt{2^6\cdot2\cdot7} = 2^3\sqrt{2\cdot7}$
$= 8\sqrt{14}$

Additional Resources

Click on the links below to download worksheets for additional practice

1. http://www.kutasoftware.com/freeia2.html Click on any of the topics in the block called "**Polynomials**", but this review concentrates mostly on the topics "**Basic shape of graphs of polynomials**" and "**Graphing polynomial functions**".

2. For in-person help, contact the Math Assistance Area. Information about our services can be found at cod.edu/academics/learning_commons/mathematics_assistance/index.aspx