DIY: *Graphing Functions: Rational Functions*

To review graphs of rational functions, watch the following set of YouTube videos explaining what rational functions are, the characteristics of their graphs, and how to sketch their graphs.

They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. Introduction: https://www.youtube.com/watch?v=L8arGNsnAIc

3. Vertical asymptotes and "holes": https://www.youtube.com/watch?v=qkUnSomHZdg

4. Horizontal asymptotes: https://www.youtube.com/watch?v=6HZ1QnNbkf0

5. Slant or Oblique asymptotes: https://www.youtube.com/watch?v=y7KuUwydVZE

6. Introduction to graphing rational functions:

   https://www.youtube.com/watch?v=LFjf22W9RBo part 1
   https://www.youtube.com/watch?v=K6YcEAMs_eU part 2
   https://www.youtube.com/watch?v=3EPxS4V1Gu0 (applies some calculus but can be understood without)

7. Graphing using transformations: https://www.youtube.com/watch?v=fvQKye9xADY
   * note that functions such as $f(x) = \frac{x-3}{x+1}$ can be graphed using transformations if the division is performed, leaving the function in the equivalent form $f(x) = 1 - \frac{4}{x+1}$. Now it can be seen that this is the basic function $y = \frac{1}{x}$, moved to the left 1 unit, stretched by a factor of 4, reflected over the x-axis, then translated up 1 unit.

Practice problems: The following problems use the techniques demonstrated in the above videos. The answers are given after the problems. Then detailed solutions, if you need them, are provided after the answer section. For further assistance and help please contact ***Math Assistance Area***.

1. Graph the following parent functions and functions with transformations from these parent functions.

a. $y = \frac{1}{x}$ b. $y = \frac{1}{x^2}$ c. $f(x) = \frac{2}{x-2}$ d. $g(x) = \frac{1}{(2x+4)} - 3$

e. $h(x) = 2 - \frac{1}{(x-1)^2}$

2. Simplify using long division, then graph using transformations: $f(x) = \frac{2x+3}{x-1}$.

3. Graph the following rational functions. Give the equations for all asymptotes and find any x- or y- intercepts.

a. $f(x) = \frac{1}{3-x}$ b. $f(x) = \frac{2x+5}{x+1}$ c. $f(x) = -\frac{1}{(x+2)^2}$

d. $f(x) = \frac{2x^2}{x^2-4}$ e. $f(x) = \frac{x^2}{x^2+1}$ f. $f(t) = -\frac{t^2+1}{t+5}$

g. $g(x) = \frac{x^2-4}{2x+2}$ h. $h(x) = \frac{x^3-4x^2+5x-3}{x-1}$ (note: this problem goes beyond the scope of the videos, but is included to show what happens to the graph when the numerator is more than 1 degree higher than the denominator.

4. Graph the following rational functions with asymptotes and/or holes (point discontinuities).

a. $f(x) = \frac{3a^2-8a+4}{2a^2-3a-2}$ b. $f(x) = \frac{x^2-16}{x-4}$ c. $g(x) = \frac{2x^3-x^2-2x+1}{x^2+3x+2}$

5. Graph the following rational functions, noting that the graph may cross a horizontal or oblique asymptote. (A graph will *never* cross a vertical asymptote.)

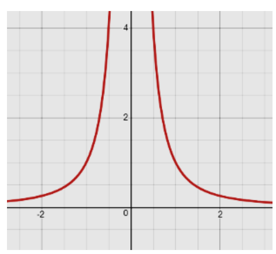
a. $f(b) = \frac{b^2-2b-8}{b^2-9}$ b. $f(x) = \frac{x}{x^2+1}$ c. $f(x) = \frac{x^3}{2x^2-8}$ d. $y = \frac{2x^3}{x^2+1}$

6. Question about asymptotes: We have seen samples of functions with only a horizontal asymptote, only an oblique asymptote, a vertical and a horizontal asymptote, and a vertical and an oblique asymptote. Can a graph of a rational function have both a horizontal and an oblique asymptote?

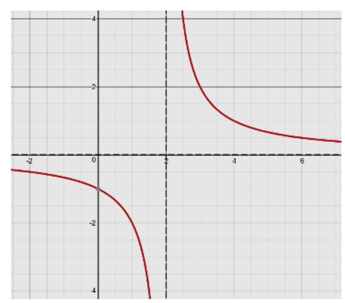
Answers to Practice Problems:

1.a.

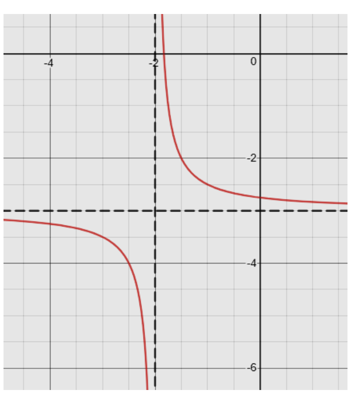
b.



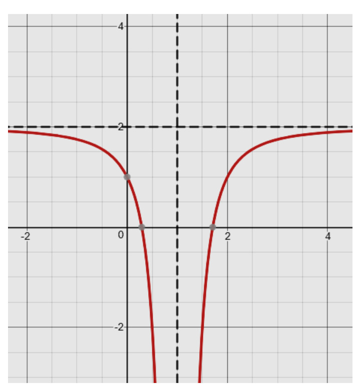
c.



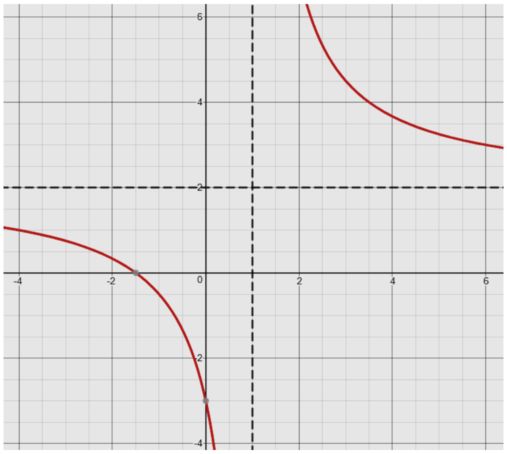
d.



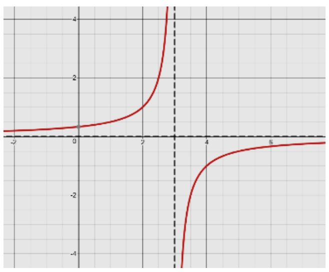
e.



2.

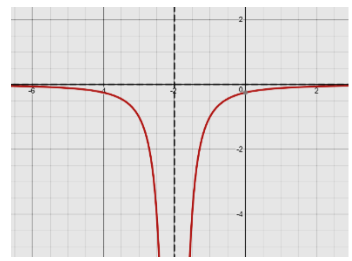


3. a.

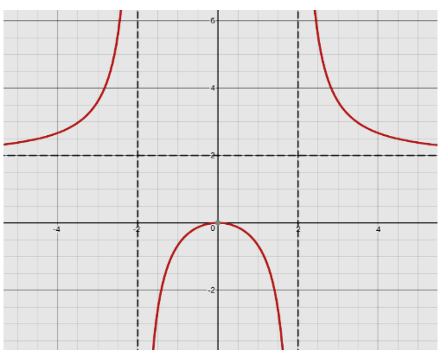


b.

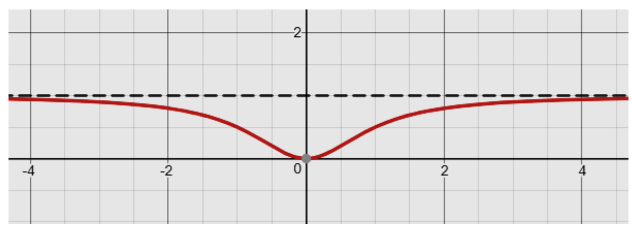
c.



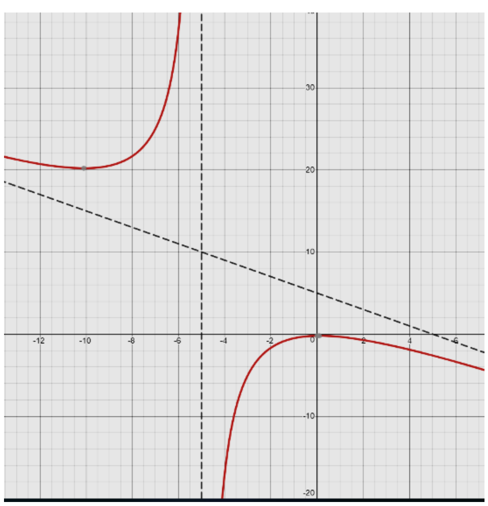
d.



e.

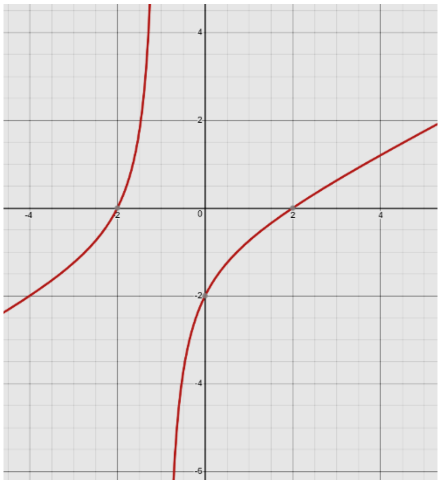


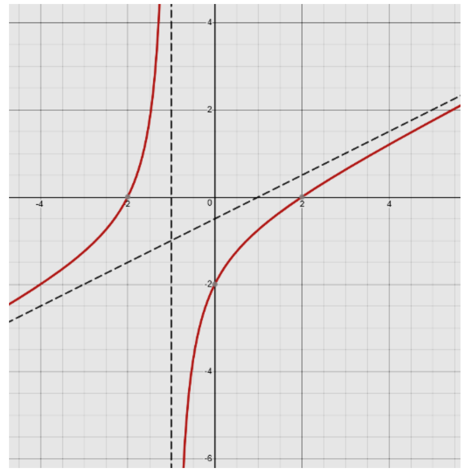
f.



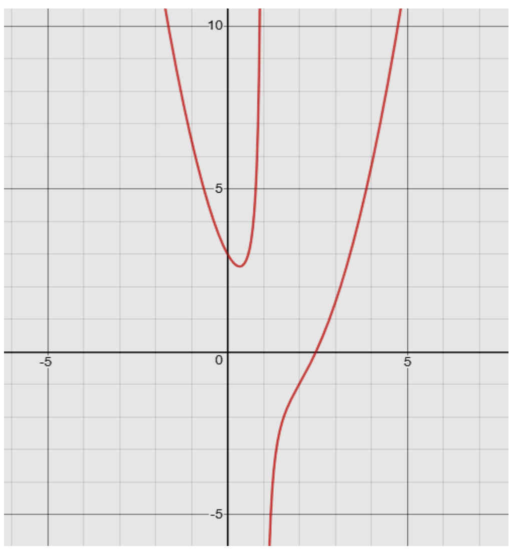
Note: The graphs of 3.g. and 3.h. are shown with/without their asymptotes for clarity.

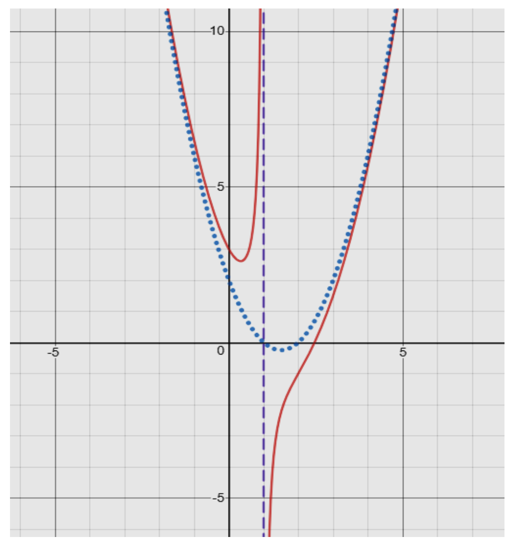
3.g.



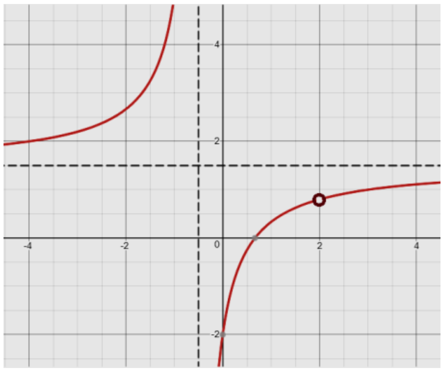


3.h.

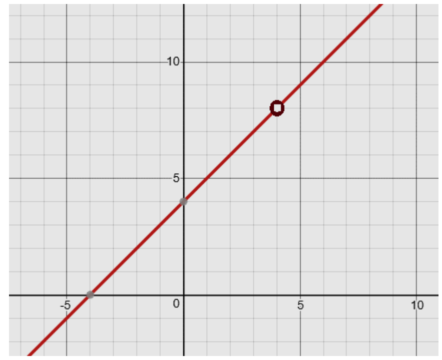




4.a.

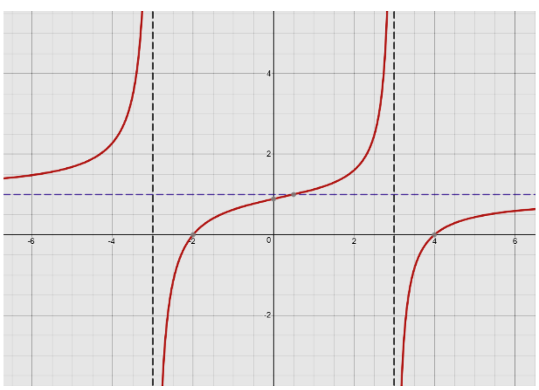


b.

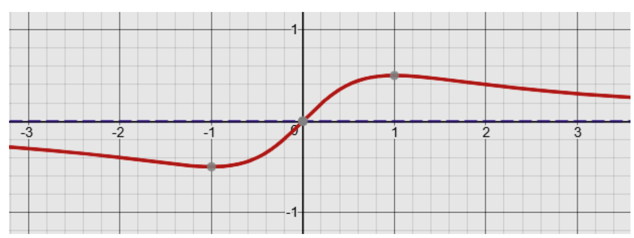


c.

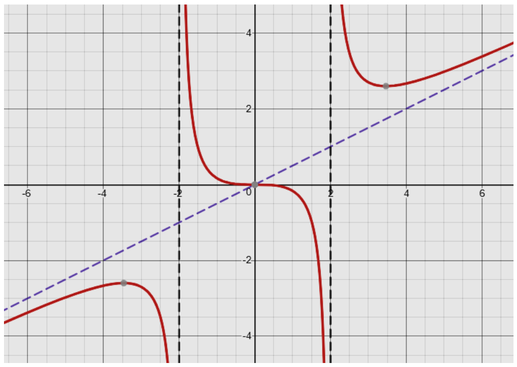
5.a.



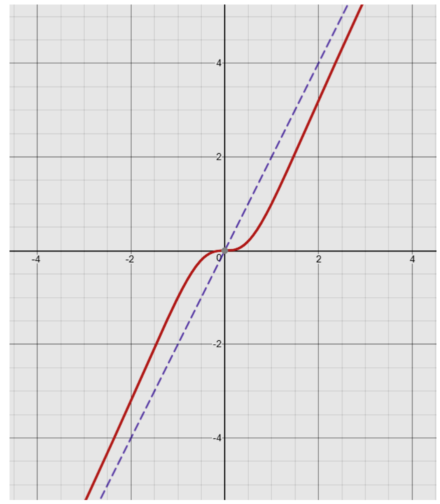
b.



5.c.



d.



6. No, unless one asymptote is only as x increases to +∞ and the other is only as x decreases to -∞. (See Detailed Solutions)

Detailed solutions are shown beginning on the following page.

Detailed Solutions:

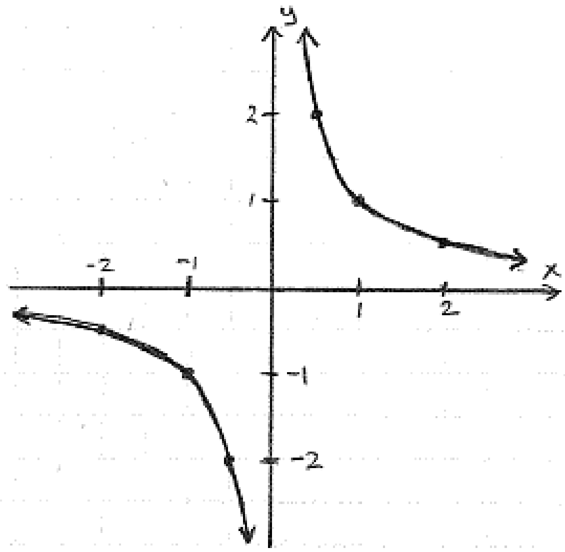
1. a. $y = \frac{1}{x}$ (parent function: reciprocal function)

has a vertical asymptote at $x=0$ → Domain: $(-\infty,0) \cup (0,\infty)$
has a horizontal asymptote at $y=0$ Range: $(-\infty,0) \cup (0,\infty)$

table of values:

x	y
−2	−1/2
−1	−1
−1/2	−2
0	(undefined).
1/2	2
1	1
2	1/2

(no x- or y- intercepts)



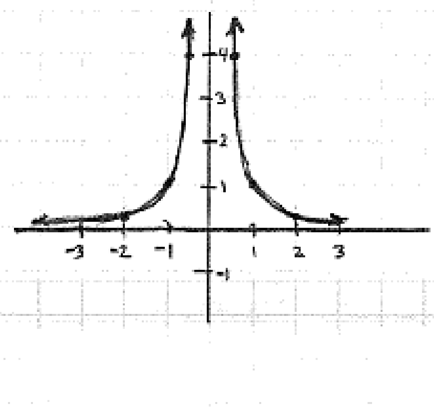
b. $y = \frac{1}{x^2}$ (parent function)

vertical asymptote: $x=0$ → Domain: $(-\infty,0) \cup (0,\infty)$
horiz. asymptote: $y=0$ Range: $(0,\infty)$

table of values:

x	y
−2	1/4
−1	1
−1/2	4
0	(undefined)
1/2	4
1	1
2	1/4

(no x- or y- intercepts).



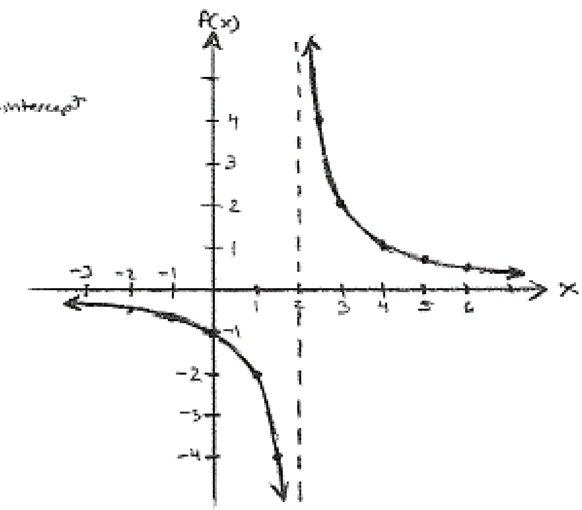
1.c. $f(x) = \dfrac{2}{x-2}$ (parent function $y = \frac{1}{x}$, translated 2 units to right, stretched away from x-axis by factor of 2.)

vertical asymptote: $x = 2$ → domain: $(-\infty, 2) \cup (2, \infty)$
horizontal asymptote: $y = 0$ Range: $(-\infty, 0) \cup (0, \infty)$

table of values:

x	f(x)
-1	$-2/3$
0	-1 ← y-intercept
1	-2
1.5	-4
2	(undefined)
2.5	4
3	2
4	1
5	$2/3$

(no x-intercept since $\frac{2}{x-2} \neq 0$ for any x)



d. $g(x) = \dfrac{1}{2x+4} - 3 = \dfrac{1}{2(x+2)} - 3$ (parent function $y = \frac{1}{x}$, translated two units to left, stretched by a factor of ½ (flattened toward x-axis), then translated down 3 units.

Vertical asymptote: $x = -2$ → domain: $(-\infty, -2) \cup (-2, \infty)$
horizontal asymptote: $y = -3$ range: $(-\infty, -3) \cup (-3, \infty)$

table of values:

x	g(x)
-4	$-3\frac{1}{4}$
-3	$-3\frac{1}{2}$
-2.5	-4
-2	(undefined)
-1.5	-2
-1	$-2\frac{1}{2}$
0	$-2\frac{3}{4}$
1	$-2\frac{5}{6}$
$-1\frac{5}{6}$	0

x-intercept (y=0)

$\frac{1}{2x+4} - 3 = 0$

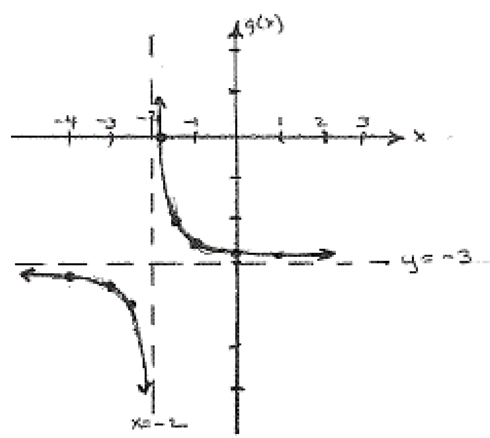
$\frac{1}{2x+4} = 3$

$1 = 3(2x+4)$

$1 = 6x + 12$

$-11 = 6x$

$x = \frac{-11}{6} = -1\frac{5}{6}$



1.e. $h(x) = 2 - \dfrac{1}{(x-1)^2}$ (parent function $y = \frac{1}{x^2}$, translated 1 unit →, reflected about x-axis, then translated 2 units ↑.)

vertical asymptote: $x=1$ → Domain: $(-\infty, 1) \cup (1, \infty)$

horizontal asymptote: $y=2$ Range: $(-\infty, 2)$

table of values:

x	$h(x)$
-2	$1\frac{8}{9}$
-1	$1\frac{3}{4}$
0	1 ← y-intercept
$\frac{1}{2}$	-2
1	(undefined)
$1\frac{1}{2}$	-2
2	1
3	$1\frac{3}{4}$
$\approx .3$	0
≈ 1.7	0

x-intercept:

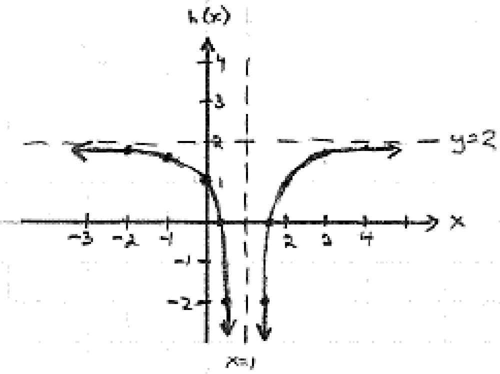
$2 - \frac{1}{(x-1)^2} = 0$

$\frac{1}{(x-1)^2} = 2$

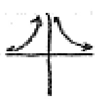
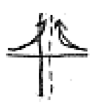
$(x-1)^2 = \frac{1}{2}$

$x - 1 = \pm\sqrt{\frac{1}{2}} \approx \pm .7$

$x \approx 1 \pm .7 \approx .3, 1.7$



graphs progressing from parent to h(x):

$y = \frac{1}{x^2}$  $y = \frac{1}{(x-1)^2}$  $y = \frac{-1}{(x-1)^2}$ 

2. $f(x) = \dfrac{2x+3}{x-1}$ →

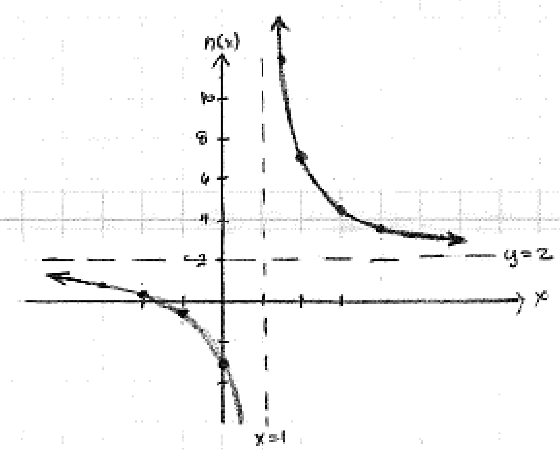
$$\begin{array}{r} 2 \\ x-1 \,\big)\, 2x+3 \\ -\underline{(2x-2)} \\ 5 \end{array}$$

$= 2 + \dfrac{5}{x-1}$ $y = \frac{1}{x}$, translated 1 unit →, stretched by a factor of 5, then translated 2 units ↑.

Vert. asymp.: $x=1$

horiz. asymp: $y=2$

x	$f(x)$
-2	$\frac{1}{3}$
-1	$-\frac{1}{2}$
0	-3
$\frac{1}{2}$	-8
1	(undefined)
$1\frac{1}{2}$	12
2	7
3	$4\frac{1}{2}$
4	$3\frac{2}{3}$



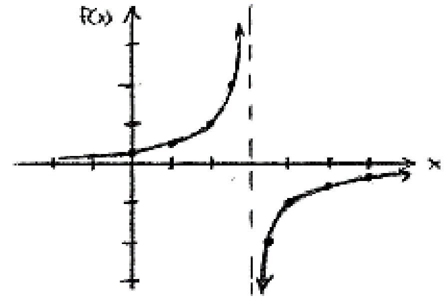
*Notes: VA = vertical asymptote
HA = horizontal asymptote

3.a. $f(x) = \frac{1}{3-x} = \frac{-1}{x-3}$

V.A.: $x=3$
H.A.: $y=0$
(reflected over x-axis)

intercepts:
if $x=0$ $y=\frac{1}{3}$

and $y \neq 0$ for any x



b. $f(x) = \frac{2x+5}{x+1}$

V.A. at $x=-1$
H.A. at $y=2$ (degree of numerator = deg. of denom. use H.A. at y = ratio of leading coefficients. $y=\frac{2}{1}$)

one way to find the horiz. asymptote (HA) is to divide each term in the numerator and denominator by the highest power of x (in this case, just x).

$$f(x) = \frac{\frac{2x}{x}+\frac{5}{x}}{\frac{x}{x}+\frac{1}{x}} = \frac{2+\frac{5}{x}}{1+\frac{1}{x}}$$

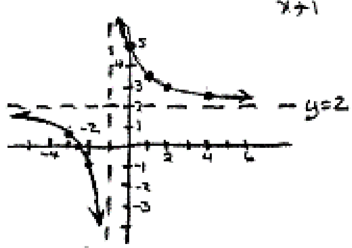
Horizontal asymptotes show what y-value the function approaches as x increases (as $x \to \infty$).

As $x \to \infty$, $\frac{1}{x} \to 0$ so

$$f(x) = \frac{2+\frac{5}{x}}{1+\frac{1}{x}} \longrightarrow \frac{2+0}{1+0} \text{ for large } |x|.$$

$y=2$ is the horiz. asymptote.

intercepts:
$x=0$ $y=5$ $(0,5)$
$y=0$ $\frac{2x+5}{x+1}=0 \to 2x+5=0$
$2x=-5$
$x=-5/2$ $\left(-\frac{5}{2},0\right)$



3.c. $f(x) = -\frac{1}{(x+2)^2}$

V.A. $x = -2$

H.A. $y = 0$ (degree of denominator > deg. of numerator)

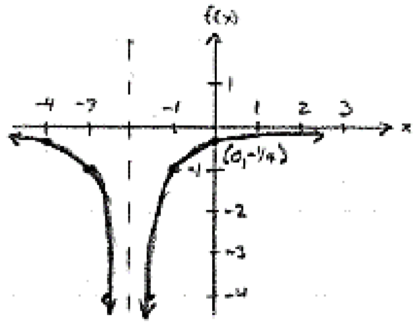
Intercepts:

$x=0 \quad y=-1/4 \quad (0, -1/4)$

(no x-intercepts)

note: this is $y = \frac{1}{x^2}$, translated 2 units ←, reflected over x-axis.

x	y
-3	-1
-2	(undefined)
-1	-1
0	$-1/4$
$-1\frac{1}{2}$	-4



d. $f(x) = \frac{2x^2}{x^2-4}$

V.A.: where $x^2-4=0$

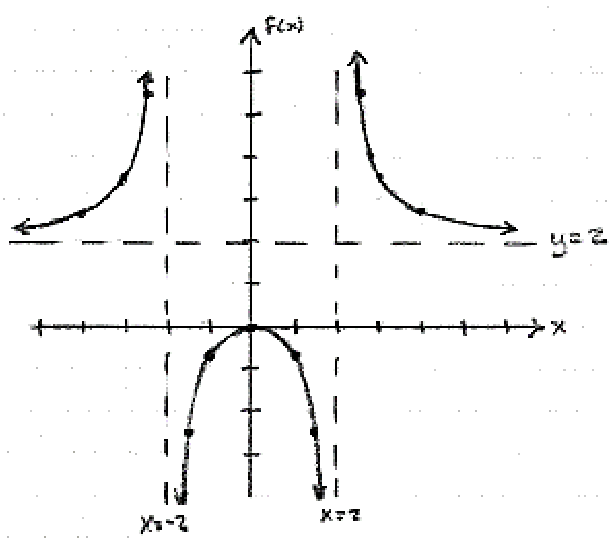
$x = -2, 2$

H.A: at $y =$ ratio of leading coefficients of numerator and denominator

$y = 2$

x	f(x)
-4	$2\frac{2}{3}$
-3	$3\frac{3}{5}$
-2	(undefined)
-1	$-\frac{2}{3}$
0	0
1	$-\frac{2}{3}$
2	(undefined)
3	$3\frac{3}{5}$
4	$2\frac{2}{3}$
± 1.5	$-2\frac{4}{7}$
± 2.5	≈ 5.6

Intercepts: $(0,0)$



3.e. $f(x) = \frac{x^2}{x^2+1}$

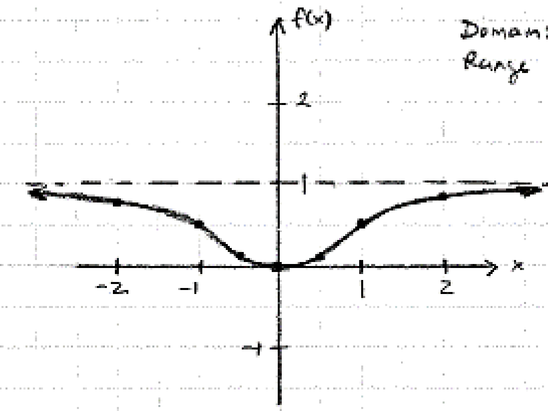
V.A: would be at x-values which make denominator = 0 but $x^2+1 \neq 0$ for any real x.
→ no vertical asymptotes.

*Note: HA can be found also by using the ratio of leading coefficients.

* H.A.: $y = \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{1}{1+\frac{1}{x^2}} \to 1$ as $x \to \infty$ $y = 1$.

Intercepts: $x=0$ $y=0$ (0,0) is only intercept

x	$f(x)$
-3	$9/10 = .9$
-2	$4/5 = .8$
-1	$\frac{1}{2} = .5$
0	0
1	$\frac{1}{2}$
2	$4/5$
$\pm 1/2$	$1/5$



Domain: $(-\infty, \infty)$
Range $[0, 1)$

f. $f(t) = -\frac{t^2+1}{t+5}$ since the numerator is one degree higher than the denominator, dividing the numerator by the denominator will show what the oblique (slant) asymptote is.

$$\begin{array}{r} -t + 5 \\ t+5 \overline{)\, -t^2 \qquad -1} \\ -(-t^2 - 5t) \\ \hline 5t \ -1 \\ -\ (5t+25) \\ \hline -26 \end{array}$$

$f(t) = -t + 5 - \frac{26}{t+5}$

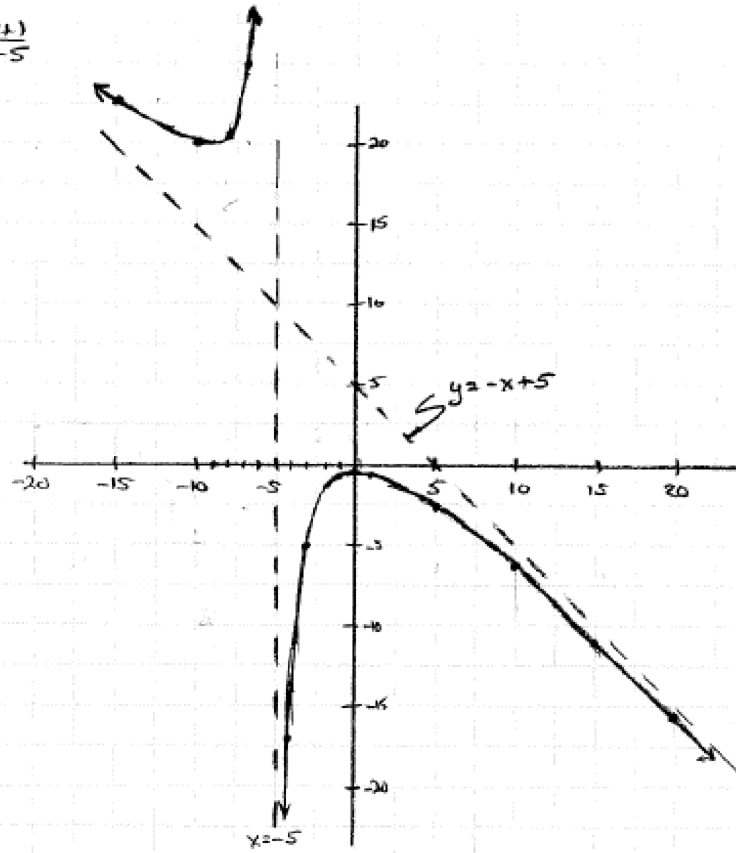
V.A $t = -5$
no horizontal asymptote.
oblique asymptote: $y = -t+5$

Intercepts: $t = 0$ $f(0) = -1/5$
if $f(t) = 0$ then $t^2+1 = 0$ (no solution – no x-intercept)

(see next page for graph)

3.f. (cont.) $f(t) = -\frac{t^2+1}{t+5}$

t	$f(t)$
-20	26.7
-15	22.6
-10	20.2
-8	21.7
-7	25
-6	37
-5	undefined
-4	-17
-3	-5
-2	$-1\frac{2}{3}$
-1	$-\frac{1}{2}$
0	$-\frac{1}{5}$
1	$-\frac{1}{3}$
5	-2.6
10	-6.7
15	-11.3
20	-16.0



g. $g(x) = \frac{x^2-4}{2x+2} = \frac{(x-2)(x+2)}{2(x+1)}$

$= \frac{1}{2}x - \frac{1}{2} - \frac{3}{2(x+1)}$

V.A. at $x=-1$

no horiz. asymptote.

Oblique asymptote: do division

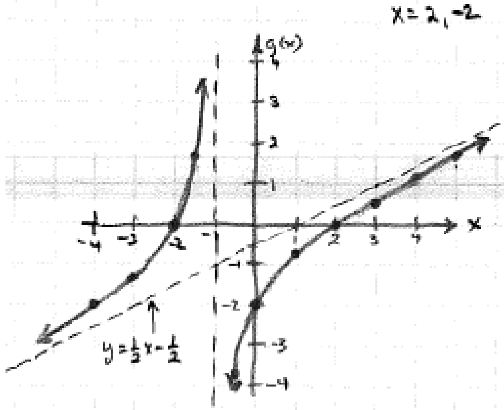
$$\begin{array}{r} \frac{1}{2}x - \frac{1}{2} \\ 2x+2 \overline{)\, x^2 + 0x - 4} \\ -(x^2 + x) \\ \hline -x - 4 \\ -(-x - 1) \\ \hline -3 \end{array}$$

intercepts: if $x=0$ $y=-2$ $(0,-2)$

if $y=0$ $x^2-4=0$

$x=2,-2$ $(2,0), (-2,0)$

Oblique asymptote: $y = \frac{1}{2}x - \frac{1}{2}$



x	y
-4	-2
-3	-1.25
-2	0
-1	(undefined)
0	-2
1	$-\frac{3}{4}$
-1.5	1.75

x	y
2	0
3	$\frac{5}{8}$
4	1.2
5	1.75
$-\frac{1}{2}$	-3.75

3.h. $h(x) = \dfrac{x^3 - 4x^2 + 5x - 3}{x-1}$

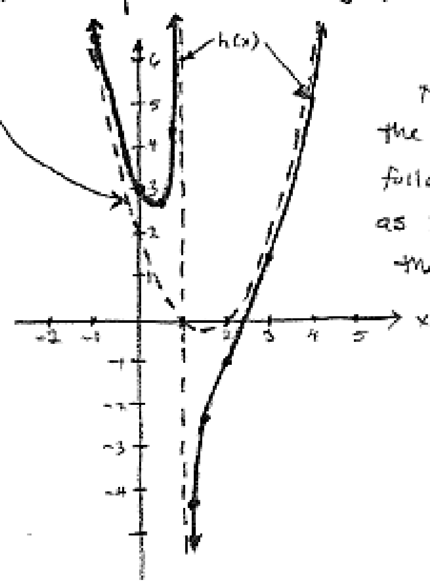
$$= x^2 - 3x + 2 - \frac{1}{x-1}$$

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x-1 & x^3 - 4x^2 + 5x - 3 \\ & -(x^3 - x^2) \\ & \quad -3x^2 + 5x \\ & \quad -(-3x^2 + 3x) \\ & \qquad 2x - 3 \\ & \qquad -(2x - 2) \\ & \qquad\quad -1 \end{array}$$

$h(x)$ has a vertical asymptote $x=1$
but no horizontal or linear oblique asymptote.

Instead, as $x \to \infty$ $y \to x^2-3x+2$ a parabola asymptote!

x	$h(x)$	$y = x^2-3x+2$
-2	$12\frac{1}{3}$	12
-1	6.5	6
0	3	2
$\frac{1}{2}$	2.75	.75
1	undefined	0
$1\frac{1}{2}$	-2.25	$-.25$
2	-1	0
3	1.5	2
4	$5\frac{2}{3}$	6
$\frac{3}{4}$	$4\frac{3}{8}$	
$1\frac{1}{4}$	-4.2	



Note how closely the graph of $h(x)$ follows the parabola as x moves away from the vertical asymptote.

Note: Graphing this function is beyond the scope of most pre-calculus courses!

4.a. $f(x) = \dfrac{3a^2 - 8a + 4}{2a^2 - 3a - 2} = \dfrac{(3a-2)\cancel{(a-2)}}{(2a+1)\cancel{(a-2)}} = \dfrac{3a-2}{2a+1}$ for $a \neq 2$

Vertical asymptote: only where $2a+1 = 0$ $\boxed{x = -\tfrac{1}{2} \leftarrow \text{V.A}}$

Hole: at $a = 2$ $f(2)$ = undefined but the remaining function, $\dfrac{3a-2}{2a+1} = \dfrac{3(2)-2}{2(2)+1} = \dfrac{4}{5}$

$\boxed{\text{hole}: (2, 4/5)}$

Horizontal asymptote: since both numerator and denom. are same degree, y = ratio of leading coefficients

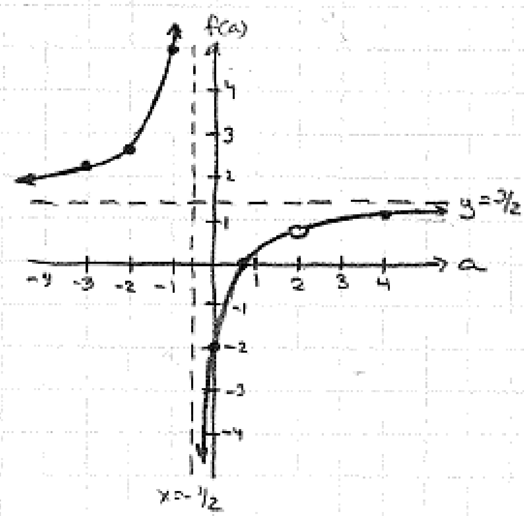
$\boxed{\text{H.A. at } y = \tfrac{3}{2}}$

Intercepts: $a = 0$ $f(0) = -2$ $\boxed{(0,-2)}$

$f(a) = 0$ when numerator $= 0 \rightarrow 3a - 2 = 0$ $a = 2/3$

$\boxed{(2/3, 0)}$

* Note: graph of $f(a)$ looks like $y = \dfrac{3a-2}{2a+1}$ except that $f(a)$ has a hole at $(2, 4/5)$

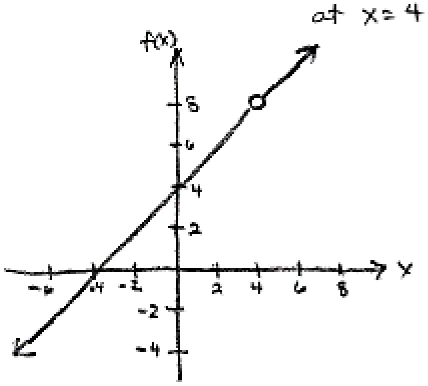


a	$f(a)$
-3	$11/5 = 2.2$
-2	$8/3 = 2.67$
-1	5
$-1/2$	undefined
0	-2
$2/3$	0
1	$1/3$
2	(would be $4/5$, but is a hole)
4	$10/9 \approx 1.1$

4b. $f(x) = \dfrac{x^2-16}{x-4} = \dfrac{(x-4)(x+4)}{(x-4)} = x+4,\ x \neq 4$

the graph of $f(x)$ is the same as the line $y = x+4$, but with a hole at $x=4$

at $x=4$, y would be $4+4=8$



(no asymptotes)

4c. $g(x) = \dfrac{2x^3-x^2-2x+1}{x^2+3x+2}$ $\longrightarrow$ factoring by grouping $2x^3-x^2-2x+1$
$= x^2(2x-1) - 1(2x-1)$
$= (2x-1)(x^2-1)$
$= (2x-1)(x-1)(x+1)$

$= \dfrac{(2x-1)(x-1)\cancel{(x+1)}}{(x+2)\cancel{(x+1)}}$ ← $x=-1$ will be a hole.

$g(x) = \dfrac{(2x-1)(x-1)}{x+2}$ or $\dfrac{2x^2-3x+1}{x+2}$ for $x \neq -1$

Intercepts:
$x=0$ $y=1/2$
$y=0$ $x=\frac{1}{2}, 1$

as $x \to -1$ $g(x) \to \dfrac{2(-1)^2-3(-1)+1}{(-1)+2} = \dfrac{2+3+1}{1} = 6$

$\boxed{\text{hole: } (-1, 6)}$

V.A.: where $x+2=0$ $\boxed{\text{V.A. at } x=-2}$

no horizontal asymptote (degree of numerator > deg. of denom)

$$\begin{array}{r|l} & 2x-7 \\ \hline x+2 & 2x^2-3x+1 \\ & -(2x^2+4x) \\ & \quad -7x+1 \\ & \quad -(-7x-14) \\ & \qquad 15 \end{array}$$

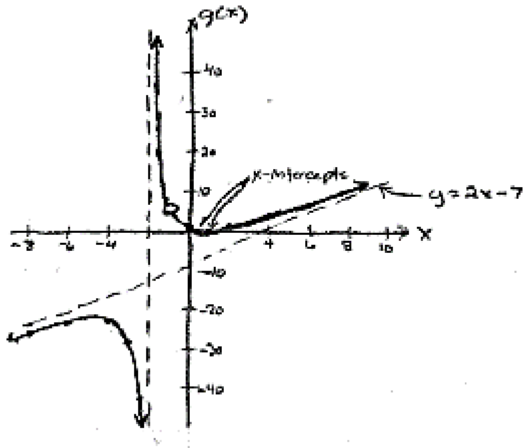
$g(x) = \underbrace{2x-7}_{} + \dfrac{15}{x+2},\ x \neq -1$

$\boxed{\text{oblique asymptote } y = 2x-7}$

(see next page for graph)

4c. (cont.)

x	$g(x)$
−8	−25.5
−6	−22.75
−4	−22.5
−3	−28
−2	(undefined)
−1	6
0	1/2
2	3/4
4	3.5
6	6.875



5a. $f(b) = \dfrac{b^2-2b-8}{b^2-9} = \dfrac{(b-4)(b+2)}{(b-3)(b+3)}$

V.A.: $b=-3$, $b=3$
H.A.: $y=1$ (ratio of leading coefficients)
(no holes, no oblique asymptote)

intercepts:

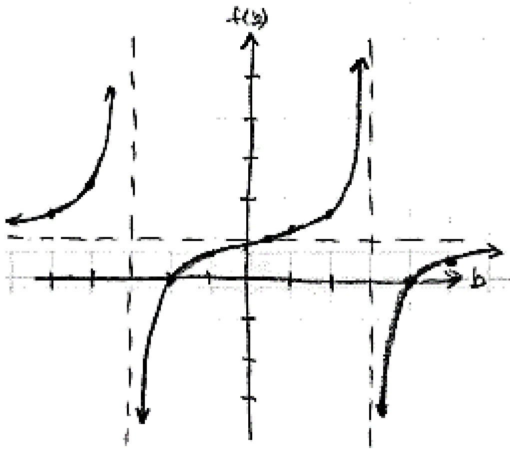
$b=0 \quad f(0) = \dfrac{-8}{-9} = \dfrac{8}{9} \quad \boxed{(0, \tfrac{8}{9})}$

from factored form, $f(b)=0$ when $b=4, -2 \rightarrow \boxed{(4,0), (-2,0)}$

To check to see if the function ever crosses its horizontal asymptote at $y=1$, set $f(b)=1$ then solve for b.

$$\frac{(b-4)(b+2)}{(b-3)(b+3)} = 1 \rightarrow \begin{aligned} b^2-2b-8 &= b^2-9 \\ -b^2 \qquad &\quad -b^2 \\ \hline -2b-8 &= -9 \\ +8 &\quad +8 \\ \hline -2b &= -1 \\ b &= 1/2 \end{aligned}$$

(yes! $f(b)$ crosses the H.A. at $b=\frac{1}{2}$).



b	$f(b)$
−5	1.7
−4	2.3
−3	V.A.
−2	0
−1	.625
0	.9
1	1.125
2	1.6
3	V.A
4	0
5	.4

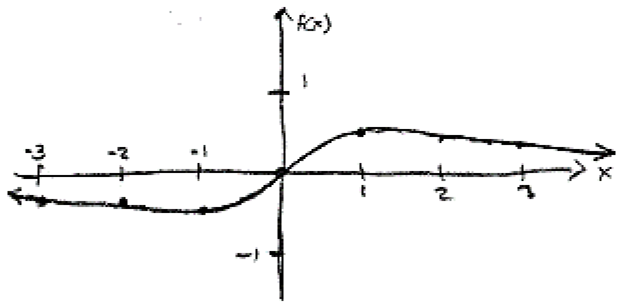
5b. $f(x) = \dfrac{x}{x^2+1}$

$\boxed{\text{V.A: none.}}$ $x^2+1 \neq 0$ for any real numbers.

$\boxed{\text{H.A: } y=0}$ (deg. of numerator < deg. of denom.)

intercepts: $x=0$ $y=0$ $\boxed{(0,0)}$

x	f(x)
−3	−.3
−2	−.4
−1	−.5
0	0
1	.5
2	.4
3	.3
−½	



5c. $f(x) = \dfrac{x^3}{2x^2-8} = \dfrac{x^3}{2(x-2)(x+2)}$

$$
\begin{array}{r}
\tfrac{1}{2}x \\
2x^2-8\,\overline{\big)\,x^3} \\
-\,(x^3-4x) \\
\hline
4x
\end{array}
$$

$= \dfrac{1}{2}x + \dfrac{4x}{2x^2-8}$

$= \dfrac{1}{2}x + \dfrac{2x}{x^2-4}$

V.A.: $x = -2, 2$

H.A.: none

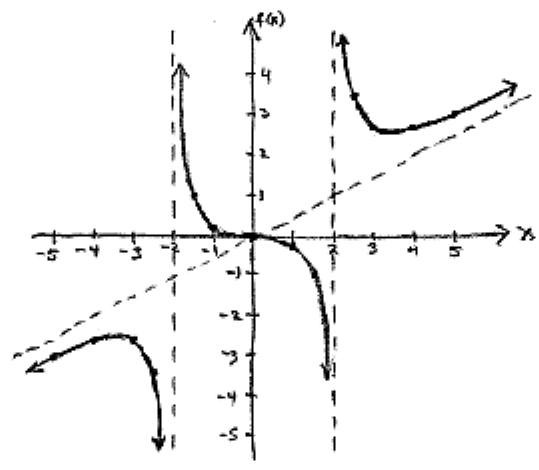
oblique asymptote: $y = \frac{1}{2}x$

For this function, to check if it's graph crosses the oblique asymptote, set $f(x) = \frac{1}{2}x$

$\cancel{\tfrac{1}{2}x} = \cancel{\tfrac{1}{2}x} + \dfrac{2x}{x^2-4}$

$\dfrac{2x}{x^2-4} = 0 \longrightarrow$ at $x=0$

(graph will cross its oblique asymptote at $x=0$).



x	f(x)
−5	−3
−4	−2.67
−3	−2.7
−2	VA
−1	.17
0	0
1	−.17
2	VA

x	f(x)
3	2.7
4	2.67
5	3.0
−2.5	−3.5
−1.5	.96
1.5	−.96
2.5	3.5

5d. $y = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

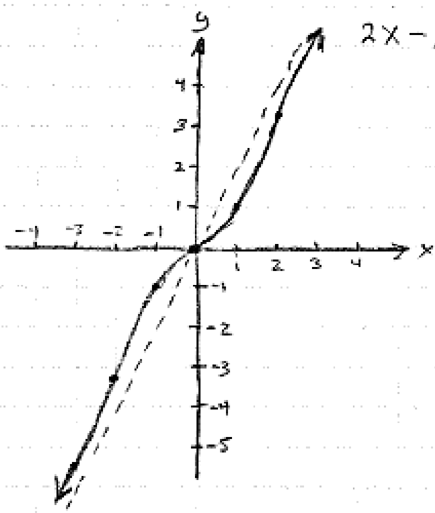
$$\begin{array}{r} 2x \\ x^2+1 \enclose{longdiv}{2x^3} \\ -(2x^3+2x) \\ \hline -2x \end{array}$$

$\boxed{\begin{array}{l} \text{V.A.: none} \\ \text{H.A.: none} \end{array}}$

oblique asymptote: $\boxed{y = 2x}$

Intercepts: $x=0$ $y=0$ $\boxed{(0,0)}$

Does the graph of y cross its oblique asymptote?



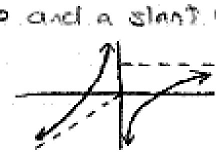
$2x - \dfrac{2x}{x^2+1} = 2x \rightarrow$ yes, at $x=0$.

x	f(x)
0	0
1	1
2	$^{16}/_{5} = 3.2$
3	$^{54}/_{10} = 5.4$
−1	−1
−2	−3.2
−3	−5.4

6. No. A graph cannot have both a horizontal and an oblique asymptote — at least not both as $x \to +\infty$ or as $x \to -\infty$.

Since these asymptotes indicate the graph's behavior for large values of x, the y-values cannot both level off to a constant value (as with a horizontal asymptote) <u>and</u> follow a slanted line.

However, there could be a function that, say, has a horizontal asymptote as $x \to +\infty$ and a slant (oblique) asymptote as $x \to -\infty$, as illustrated here:



(this would be a piecewise defined function)

Additional Resources

Click on the links below to download worksheets for additional practice

1. http://www.kutasoftware.com/freeia2.html In the block entitled "Rational Expressions", click on the first two topics in the block-- "**Graphing Simple Rational Functions**" and "**Graphing General Rational functions**".

2. For in-person help, contact the Math Assistance Area. Information about our services can be found at https://cod.edu/academics/learning_commons/mathematics_assistance/index.aspx