DIY: *Radicals and Rational Exponents*

To review Radicals and Rational Exponents, watch the following set of YouTube videos. (Note for your security: all hyperlink addresses should begin with https://youtube.com/watch?v=) They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end.

1. https://www.youtube.com/watch?v=B4zejSI8zho Exponents and Radicals. An introduction to radicals.

2. https://www.youtube.com/watch?v=1EO6rqtSTgU How to Simplify Square Roots using a Factor Tree.

   Note: At 2:49 in this video, the moderator incorrectly states that $\sqrt{5\cdot5\cdot3\cdot3} = 5\cdot3\cdot\sqrt{0}$
   The correct statement should be that $\sqrt{5\cdot5\cdot3\cdot3} = 5\cdot3\cdot\sqrt{1} = 5\cdot3\cdot1 = 15$

3. https://www.youtube.com/watch?v=5wCldWohDIg Simplifying numerical roots

   Note: At 7:11, in example 3, the moderator wrote the answer as $3\sqrt{2}$. The correct answer is $\mathbf{3\sqrt[3]{2}}$

4. https://www.youtube.com/watch?v=8gZQTbg4pWE Simplifying higher powered radicals

5. https://www.youtube.com/watch?v=oqexEBWJM_g Adding and multiplying square roots

6. https://www.youtube.com/watch?v=LfsJxeye1CA Dividing radicals and Rationalizing denominators

7. https://www.youtube.com/watch?v=JW5VZrMXANk Rationalizing denominators with variables

8. https://www.youtube.com/watch?v=lZfXc4nHooo Rational Exponent

9. https://www.youtube.com/watch?v=GipavLCnke0 More on rational exponents

   Note: The moderator states that $\sqrt[3]{64^2} = (\sqrt[3]{64})^2$ or $(64^{\frac{1}{3}})^2$. Using the actual order of operations, $\sqrt[3]{64^2} = (64^2)^{1/3}$. However, the two are equal and it is easier to take the cube root first, then square.

8. https://www.youtube.com/watch?v=qzkUgkcuSvA Reducing the index of a radical

   Note: At the end of this video, the moderator says it would be considered incorrect if the square root of x were written as $\sqrt[2]{x}$. This is not incorrect, but by convention, the index is not usually written for square roots.

Practice Problems:

1. Complete the following:

$1^2 =$ ____, so $\sqrt{}$ = 1 $\qquad$ $2^2 =$ ____, so $\sqrt{}$ = 2 $\qquad$ $3^2 =$ ____, so $\sqrt{}$ = 3

$4^2 =$ ____, so $\sqrt{\ \ }$ = 4 $5^2 =$ ____, so $\sqrt{\ \ }$ = 5 $6^2 =$ ____, so $\sqrt{\ \ }$ = 6

$7^2 =$ ____, so $\sqrt{\ \ }$ = 7 $8^2 =$ ____, so $\sqrt{\ \ }$ = 8 $9^2 =$ ____, so $\sqrt{\ \ }$ = 9

$10^2 =$ ____, so $\sqrt{\ \ }$ = 10 $11^2 =$ ____, so $\sqrt{\ \ }$ = 11 $12^2 =$ ____, so $\sqrt{\ \ }$ = 12

$13^2 =$ ____, so $\sqrt{\ \ }$ = 13 $14^2 =$ ____, so $\sqrt{\ \ }$ = 14 $15^2 =$ ____, so $\sqrt{\ \ }$ = 15

$20^2 =$ ____, so $\sqrt{\ \ }$ = 20 $25^2 =$ ____, so $\sqrt{\ \ }$ = 25 $0^2 =$ ____, so $\sqrt{\ \ }$ = 0

2. a. It would NOT be correct to say that $(-3)^2 = 9$, so $\sqrt{9}$ = (-3). Why is this incorrect?

   b. For what values of **a** would it be correct to say $(a)^2 = a^2$, so $\sqrt{a^2} = a$?

3. Complete the following:

   $2^3 =$ _____, so ______ = 2 $3^4 =$ _____, so ______ = 3 $4^3 =$ _____, so ______ = 4

   $5^6 =$ _____, so ______ = 5 $10^5 =$ _____, so ______ = 10 $1^{10} =$ _____, so ______ = 1

4. a. It IS be correct to say that $(-6)^3 = -216$, so $\sqrt{-216}$ = (-6). Why is this correct for cube roots but not for square roots?

   b. For what values of **a** would it be correct to say $(a)^3 = a^3$, so $\sqrt[3]{a^3} = a$?

5. Simplify: (do not use a calculator)

   a. $\sqrt{576} =$ _______ b. $\sqrt{63} =$ ______ c. $\sqrt{432} =$ _______ d. $\sqrt{245} =$ ______

6. Simplify the following:

a. $\sqrt[4]{324} =$ _______ b. $\sqrt[5]{3^5} =$ ______ c. $\sqrt[3]{2^3 3^6} =$ ______ d. $\sqrt[3]{4^6 \cdot 5^8} =$ _____

e. $\sqrt[4]{4^6 \cdot 5^8} =$ _____ f. $\sqrt[3]{408{,}240} =$ ______ g. $\sqrt{-576} =$ ________

7. Note: These problems involve variables. Since, unless stated otherwise, a variable may take on positive or negative values, it is necessary to make the following distinctions:

1. If the index n is **even**, then $\sqrt[n]{x^n} = |x|$. The absolute value of x, written as $|x|$, is defined as

$$|x| = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$$

2. If the index n is **odd**, then $\sqrt[n]{x^n} = x$

a. $\sqrt[3]{a^3 b^{12}} =$ ______ b. $\sqrt{x^6} =$ ______ c. $\sqrt[3]{y^5} =$ ______ d. $\sqrt[4]{k^6}, k > 0 =$ ______

8. Simplify: (assume all variables are > 0)

a. $\sqrt{12xy^3} \cdot \sqrt{3x^3y^4} =$ __________ b. $\sqrt[3]{4a^5b^2c} \cdot \sqrt[3]{4a^4b^2c^2} =$ _______________

9. Simplify: (assume all variables are > 0)

a. $\dfrac{\sqrt{5x^7y^2}}{\sqrt{15x^4y^5}} =$ __________ b. $\sqrt[3]{\dfrac{5x^5}{6w^8}} =$ ______________

10. Simplify: (assume all variables are > 0)

a. $5\sqrt{40} + 6\sqrt{10} - 3\sqrt{90} =$ _________ b. $3\sqrt{x^5} - 4\sqrt{x^4} + x\sqrt{16x^3} =$ ____________

11. Rewrite in rational exponent form: a. $\sqrt[5]{x^2} =$ ______ b. $\left(\sqrt[3]{b}\right)^5$

12. Rewrite in radical form: a. $x^{\frac{4}{7}} =$ ________ b. $(4v)^{\frac{3}{2}}$

13. Combine the following *unlike radicals* by first writing in rational exponent form, combine using exponent rules, then rewrite using a single radical. Simplify if possible.

a. $\sqrt{x} \cdot \sqrt[3]{x} =$ ______ b. $\sqrt[3]{4a^5} \cdot \sqrt[4]{32a^9} =$ _______

14. Reduce the index in the following expression. (It will help to rewrite in fractional exponent form and using prime factors.)

a. $3\sqrt[4]{4} =$ _____ b. $\sqrt[6]{27x^3} =$ ______ c. $\sqrt[n]{y^{3n}} =$ ______ d. $\sqrt[6]{256} =$ ______

15. Perform the indicated operations and simplify the result:

a. $\sqrt{3}(6+\sqrt{6})$ b. $(\sqrt{5}-\sqrt{2})(\sqrt{5}+3)$ c. $(2+\sqrt{3})^2$

16. Simplify, leaving only rational denominators:

a. $\frac{2}{4-\sqrt{2}}$ b. $\frac{3+\sqrt{5}}{3-\sqrt{5}}$ c. $\frac{\sqrt{2}}{x+\sqrt{2}}$

Answers:

1. (see Detailed Solutions) 2. a. (see Detailed Solutions) b. $a \geq 0$

3. (see Detailed Solutions) 4. a. (see Detailed Solutions) b. a = any real number

5. a. 24 b. $3\sqrt{7}$ c. $12\sqrt{3}$ d. $7\sqrt{5}$

6. a. $3\sqrt[4]{4}$ (can be simplified. See 14. a.) b. 3 c. 18 d. $8000\sqrt[3]{25}$

e. 200 f. $18\sqrt[3]{70}$ g. not a real number

7. a. ab^4 b. $|x^3|$ *or* $|x|^3$ c. $y\sqrt[3]{y^2}$ d. $k\sqrt[4]{k^2} = k\sqrt{k}$

8. a. $6x^2y^3\sqrt{y}$ b. $2a^3bc\sqrt[3]{2b}$ 9. a. $\frac{x\sqrt{3xy}}{3y^2}$ b. $\frac{x\sqrt[3]{180x^2w}}{6w^3}$

10. a. $7\sqrt{10}$ b. $7x^2\sqrt{x} - 4x^2$ 11. a. $x^{\frac{2}{5}}$ b. $b^{\frac{5}{3}}$

12. a. $\sqrt[7]{x^4}$ b. $\sqrt{(4v)^3} = 8v\sqrt{v}$ 13. a. $\sqrt[6]{x^5}$ b. $2a^3\ \sqrt[12]{2^{11}a^{11}} = 2a^3\ \sqrt[12]{2048a^{11}}$

14. a. $3\sqrt{2}$ b. $\sqrt{3x}$ c. y^3 d. $2\sqrt[3]{2}$

15. a. $6\sqrt{3} + 3\sqrt{2}$ b. $5 + 3\sqrt{5} - \sqrt{10} - 3\sqrt{2}$ c. $7 + 4\sqrt{3}$

16. a. $\frac{4+\sqrt{2}}{7}$ b. $\frac{7+3\sqrt{5}}{2}$ c. $\frac{x\sqrt{2}-2}{x^2-2}$

(Detailed Solutions beginning on the next page)

Detailed Solutions

1. $1^2 = 1$, so $\sqrt{1} = 1$ $\quad 2^2 = 4$, so $\sqrt{4} = 2$ $\quad 3^2 = 9$, so $\sqrt{9} = 3$

$4^2 = 16$, so $\sqrt{16} = 4$ $\quad 5^2 = 25$, so $\sqrt{25} = 5$ $\quad 6^2 = 36$, so $\sqrt{36} = 6$

$7^2 = 49$, so $\sqrt{49} = 7$ $\quad 8^2 = 64$, so $\sqrt{64} = 8$ $\quad 9^2 = 81$, so $\sqrt{81} = 9$

$10^2 = 100$ so $\sqrt{100} = 10$ $\quad 11^2 = 121$, so $\sqrt{121} = 11$ $\quad 12^2 = 144$, so $\sqrt{144} = 12$

$13^2 = 169$, so $\sqrt{169} = 13$ $\quad 14^2 = 196$, so $\sqrt{196} = 14$ $\quad 15^2 = 225$, so $\sqrt{225} = 15$

$20^2 = 400$, so $\sqrt{400} = 20$ $\quad 25^2 = 625$, so $\sqrt{625} = 25$ $\quad 0^2 = 0$, so $\sqrt{0} = 0$

note: it is helpful to memorize these perfect square numbers.

2. a. $(-3)^2 = 9$ but $\sqrt{9}$ is defined as the <u>principle</u> square root of 9, which is 3.

b. $\sqrt{a^2} = a$ <u>only</u> for non-negative values of a.

In fact, the absolute value of a, written as $|a|$ is defined as $|a| = \sqrt{a^2}$

so $|-3| = \sqrt{(-3)^2} = \sqrt{9} = 3$

3. $2^3 = 8$, so $\sqrt[3]{8} = 2$ $\quad 3^4 = 81$ so $\sqrt[4]{81} = 3$ $\quad 4^3 = 64$ so $\sqrt[3]{64} = 4$

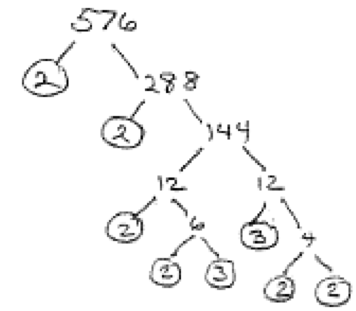
$5^6 = 15,626$ so $\sqrt[6]{15,626} = 5$ $\quad 10^5 = 100,000$, so $\sqrt[5]{100,000} = 10$

$1^{10} = 1$ so $\sqrt[10]{1} = 1$

4. a. Saying $\sqrt[3]{-216} = -6$ is correct. Since a negative number, raised to an <u>odd</u> power is still negative, radicals with an <u>odd</u> index can be used with negative radicands.

b. $(a)^3 = a^3$, so $\sqrt[3]{a^3} = a$ is correct for all real numbers a.

5. a. $\sqrt{576}$ Use a factor tree to find the prime factorization of 576



$$576 = 2^6 \cdot 3^2$$

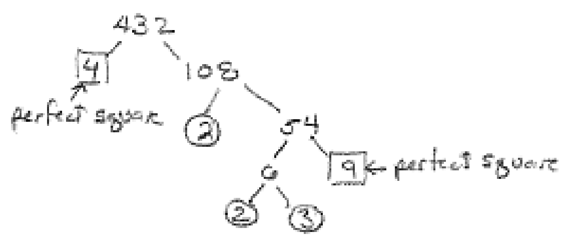
$$\sqrt{576} = \sqrt{2^6 \cdot 3^2} = \sqrt{2^6} \cdot \sqrt{3^2}$$
$$= 2^3 \cdot 3 = 8 \cdot 3 = 24$$

checking:

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array} \checkmark$$

b. $\sqrt{63} = \sqrt{9 \cdot 7} = \sqrt{9} \cdot \sqrt{7} = \boxed{3\sqrt{7}}$ (if you can spot perfect square factors in a large number, then it is not necessary to do a factor tree).

c. $\sqrt{432}$



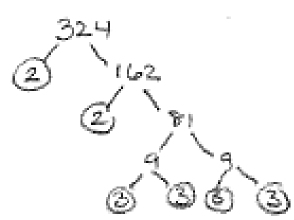
$$= \sqrt{4 \cdot 2^2 \cdot 3 \cdot 9}$$
$$= 2 \cdot 2 \cdot 3\sqrt{3} = \boxed{12\sqrt{3}}$$

checking: $(12\sqrt{3})^2 = 12^2 \cdot \sqrt{3}^2 = 144 \cdot 3 = 432$

d. $\sqrt{245}$

$$= \sqrt{5 \cdot 49} = \boxed{7\sqrt{5}}$$

6. a. $\sqrt[4]{324}$



$$= \sqrt[4]{2^2 \cdot 3^4} = 3\sqrt[4]{2^2}$$
$$= \boxed{3\sqrt[4]{4}} \; *$$

*note: the index on this can be reduced. See problem 14.a.

6.b. $\sqrt[5]{3^5} = 3$

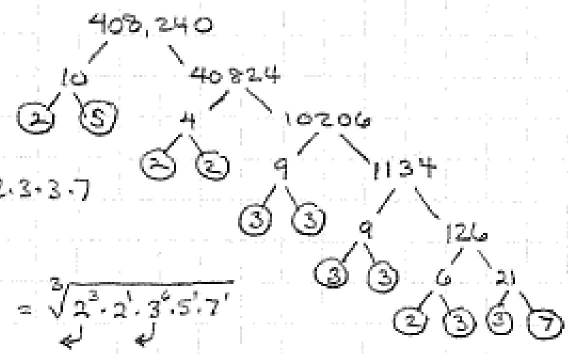
c. $\sqrt[3]{2^3\cdot 3^6} = 2\cdot 3^2$
$= 2\cdot 9 = 18$

divide exponents by index to bring that factor out of $\sqrt{\ }$

d. $\sqrt[3]{4^6\cdot 5^8}$
$= \sqrt[3]{4^6\cdot 5^6\cdot 5^2}$
$= 4^3\cdot 5^3\sqrt[3]{5^2}$
$= 64(125)\sqrt[3]{25}$
$= \boxed{8000\sqrt[3]{25}}$

e. $\sqrt[4]{4^6 5^8} = \sqrt[4]{4^4\cdot 4^2\cdot 5^8} = 4\cdot 5^2\cdot\sqrt[4]{4^2}$ but $4^2 = (2^2)^2 = 2^4$
$= 4\cdot 25\cdot\sqrt[4]{2^4}$
$= 4\cdot 25\cdot 2$
$= \boxed{200}$

f. $\sqrt[3]{408,240}$



$408,240 =$
$2\cdot5\cdot2\cdot2\cdot3\cdot3\cdot3\cdot3\cdot2\cdot3\cdot3\cdot7$
$= 2^4 3^6\cdot5\cdot7$

$\sqrt[3]{408,240} = \sqrt[3]{2^4\cdot3^6\cdot5\cdot7} = \sqrt[3]{2^3\cdot2^1\cdot3^6\cdot5^1\cdot7^1}$
$= 2\cdot3^2\sqrt[3]{2\cdot5\cdot7} = 2\cdot9\sqrt[3]{70} = 18\sqrt[3]{70}$

Check: $(18^3)(70) = 5832\cdot70 = 408,240$ ✓

g. $\sqrt{-576}$ is not a real number.

7 a. $\sqrt[3]{a^3 b^{12}} = \boxed{ab^4}$ b. $\sqrt{x^6} = \boxed{|x^3| \text{ or } |x|^3}$ c. $\sqrt[3]{y^5} = \sqrt[3]{y^3\cdot y^2}$
$= \boxed{y\sqrt[3]{y^2}}$

d. $\sqrt[4]{k^6},\ k>0 = \sqrt[4]{k^4\cdot k^2} = \boxed{k\sqrt[4]{k^2}}$ but the index can be reduced since $\sqrt[4]{k^2} = k^{\frac{2}{4}} = k^{\frac{1}{2}} = \sqrt{k}$
$= \boxed{k\sqrt{k}}$
(absolute value not needed since it is given that $k>0$).

8. a. $\sqrt{12xy^3}\cdot\sqrt{3x^3y^4} = \sqrt{12xy^3\cdot 3x^3y^4} = \sqrt{36x^4y^7}$

$= \sqrt{6^2\cdot x^4\cdot y^6\cdot y^1} = \boxed{6x^2y^3\sqrt{y}}$

b. $\sqrt[3]{4a^5b^2c}\cdot\sqrt[3]{4a^4b^2c^2} = \sqrt[3]{\underset{2^4}{16}\,a^9b^4c^3} = \sqrt[3]{2^3\cdot 2^1\cdot a^9\cdot b^3\cdot b^1\cdot c^3}$

$= \boxed{2a^3bc\sqrt[3]{2b}}$

9. a. $\dfrac{\sqrt{5x^7y^2}}{\sqrt{15x^4y^5}} = \sqrt{\dfrac{5x^7y^2}{15x^4y^5}} = \sqrt{\dfrac{5}{15}\cdot\dfrac{x^7}{x^4}\cdot\dfrac{y^2}{y^5}} = \sqrt{\dfrac{x^3}{3y^3}} = \dfrac{\sqrt{x^2\cdot x}}{\sqrt{3\cdot y^2\cdot y}}$

$= \dfrac{x\sqrt{x}}{y\sqrt{3y}} = \dfrac{x\sqrt{x}}{y\sqrt{3y}}\cdot\dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{x\sqrt{3xy}}{y\cdot 3y} = \boxed{\dfrac{x\sqrt{3xy}}{3y^2}}$

now must rationalize denominator

b. $\sqrt[3]{\dfrac{5x^5}{6w^8}} = \dfrac{\sqrt[3]{5\cdot x^3\cdot x^2}}{\sqrt[3]{2\cdot 3\cdot w^6\cdot w^2}} = \dfrac{x\sqrt[3]{5x^2}}{w^2\sqrt[3]{2\cdot 3\cdot w^2}}\cdot\dfrac{\sqrt[3]{2^2 3^2 w}}{\sqrt[3]{2^2 3^2 w}} = \dfrac{x\sqrt[3]{2^2 3^2\cdot 5\cdot x^2\cdot w}}{w^2\sqrt[3]{2^3\cdot 3^3\cdot w^3}}$

to rationalize this denominator, all factors must be perfect cubes.

$= \dfrac{x\sqrt[3]{4\cdot 9\cdot 5x^2w}}{w^2\cdot 2\cdot 3\cdot w} = \boxed{\dfrac{x\sqrt[3]{180x^2w}}{6w^3}}$

10. a. $5\sqrt{40} + 6\sqrt{10} - 3\sqrt{90} = 5\sqrt{4\cdot 10} + 6\sqrt{10} - 3\sqrt{9\cdot 10}$

$= 5\cdot 2\sqrt{10} + 6\sqrt{10} - 3\cdot 3\sqrt{10} = 10\sqrt{10} + 6\sqrt{10} - 9\sqrt{10}$

$= (10+6-9)\sqrt{10} = \boxed{7\sqrt{10}}$

b. $3\sqrt{x^5} - 4\sqrt{x^4} + x\sqrt{16x^3} = 3\sqrt{x^4\cdot x} - 4\sqrt{x^4} + 4x\sqrt{x^2\cdot x}$

$= 3x^2\sqrt{x} - 4x^2 + 4x^2\sqrt{x} = \boxed{7x^2\sqrt{x} - 4x^2}$

these are like terms

these are not like terms

11. a. $\sqrt[5]{x^2} = x^{\frac{2}{5}}$

numerator of exponent

denominator of exponent

b. $\left(\sqrt[3]{b}\right)^5$ is the same as $\sqrt[3]{b^5}$

$= b^{\frac{5}{3}}$

12. a. $x^{\frac{4}{7}} = \sqrt[7]{x^4}$

b. $(4v)^{3/2} = \boxed{\sqrt{(4v)^3}}$

this could be simplified

$= \sqrt{(2^2)^3 v^3} = \sqrt{2^6 v^2 \cdot v^1}$

$= 2^3 \cdot v\sqrt{v} = \boxed{8v\sqrt{v}}$

13. a. $\sqrt{x}\cdot\sqrt[3]{x} = x^{\frac{1}{2}}\cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\frac{3}{6}+\frac{2}{6}} = x^{\frac{5}{6}} = \boxed{\sqrt[6]{x^5}}$

b. $\sqrt[3]{4a^5}\cdot\sqrt[4]{32a^9} = \sqrt[3]{2^2\cdot a^3\cdot a^2}\cdot\sqrt[4]{2^5\cdot a^8\cdot a} = a\sqrt[3]{2^2\cdot a^2}\cdot a^2\sqrt[4]{2^4\cdot 2a}$

$= a\sqrt[3]{2^2a^2}\cdot 2a^2\sqrt[4]{2a} = 2a^3\sqrt[3]{2^2a^2}\cdot\sqrt[4]{2a}$

$= 2a^3\left[2^{2/3}a^{2/3}\cdot 2^{1/4}\cdot a^{1/4}\right] = 2a^3\left[2^{\frac{2}{3}+\frac{1}{4}}\cdot a^{\frac{2}{3}+\frac{1}{4}}\right]$

$= 2a^3\left[2^{\frac{8}{12}+\frac{3}{12}}\cdot a^{\frac{8}{12}+\frac{3}{12}}\right] = 2a^3\left(2^{\frac{11}{12}}\cdot a^{\frac{11}{12}}\right) = \boxed{2a^3\sqrt[12]{2^{11}a^{11}}}$

or $\boxed{2a^3\sqrt[12]{2048a^{11}}}$

14. a. $3\sqrt[4]{4} = 3\sqrt[4]{2^2} = 3\cdot 2^{\frac{2}{4}} = 3\cdot 2^{\frac{1}{2}} = \boxed{3\sqrt{2}}$

b. $\sqrt[6]{27x^3} = \sqrt[6]{3^3x^3} = 3^{\frac{3}{6}}\cdot x^{\frac{3}{6}} = 3^{\frac{1}{2}}x^{\frac{1}{2}} = \boxed{\sqrt{3x}}$

c. $\sqrt[n]{y^{3n}} = y^{\frac{3n}{n}} = \boxed{y^3}$

d. $\sqrt[6]{256} = \sqrt[6]{2^8} = \sqrt[6]{2^6\cdot 2^2} = 2\sqrt[6]{2^2} = 2\cdot 2^{\frac{2}{6}} = 2\cdot 2^{\frac{1}{3}} = \boxed{2\sqrt[3]{2}}$

15. a. $\sqrt{3}(6+\sqrt{6}) = \sqrt{3}\cdot 6 + \sqrt{3}\cdot\sqrt{6}$
$= 6\sqrt{3} + \sqrt{18} = 6\sqrt{3} + \sqrt{2\cdot 3^2}$
$= \boxed{6\sqrt{3} + 3\sqrt{2}}$

(these are unlike radicals, so cannot be combined)

b. $(\sqrt{5}-\sqrt{2})(\sqrt{5}+3) = \sqrt{5}\cdot\sqrt{5} + 3\sqrt{5} - \sqrt{2}\sqrt{5} - 3\sqrt{2}$
$= \boxed{5 + 3\sqrt{5} - \sqrt{10} - 3\sqrt{2}}$

(all radicals are simplified and all are unlike terms, so no terms can be combined.)

c. $(2+\sqrt{3})^2 = (2+\sqrt{3})(2+\sqrt{3}) = 2\cdot 2 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{3}\cdot\sqrt{3}$
$= 4 + 4\sqrt{3} + 3$
$= \boxed{7 + 4\sqrt{3}}$

16. a. $\dfrac{2}{4-\sqrt{2}} = \dfrac{2}{(4-\sqrt{2})}\cdot\dfrac{(4+\sqrt{2})}{(4+\sqrt{2})}$ (multiply both numerator and denominator by the conjugate of the denominator.)

$= \dfrac{2(4+\sqrt{2})}{4\cdot 4 + \cancel{4\sqrt{2}} - \cancel{4\sqrt{2}} - \sqrt{2}\cdot\sqrt{2}} = \dfrac{8+2\sqrt{2}}{16-2} = \dfrac{8+2\sqrt{2}}{14}$

$= \dfrac{\cancel{2}^{1}(4+\sqrt{2})}{\cancel{14}_{7}} = \boxed{\dfrac{4+\sqrt{2}}{7}}$

b. $\dfrac{3+\sqrt{5}}{3-\sqrt{5}} = \dfrac{(3+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \dfrac{9+3\sqrt{5}+3\sqrt{5}+\sqrt{5}\cdot\sqrt{5}}{9+\cancel{3\sqrt{5}}-\cancel{3\sqrt{5}}-\sqrt{5}\cdot\sqrt{5}}$

$= \dfrac{9+6\sqrt{5}+5}{9-5} = \dfrac{14+6\sqrt{5}}{4} = \dfrac{\cancel{2}^{1}(7+3\sqrt{5})}{\cancel{4}_{2}}$

$= \boxed{\dfrac{7+3\sqrt{5}}{2}}$

-7-

16. c. $\frac{\sqrt{2}}{x+\sqrt{2}} = \frac{\sqrt{2}(x-\sqrt{2})}{(x+\sqrt{2})(x-\sqrt{2})} = \frac{\sqrt{2}(x-\sqrt{2})}{x^2-\cancel{\sqrt{2}x}+\cancel{\sqrt{2}x}-\sqrt{2}\sqrt{2}}$

$= \boxed{\frac{\sqrt{2}(x-\sqrt{2})}{x^2-2}}$ or $\boxed{\frac{x\sqrt{2}-2}{x^2-2}}$

note: the (-2)'s do not cancel.

Additional Resources

For more practice, the following are free web sources:

1. https://www.kutasoftware.com/free.html Choose the Radical Expressions block.

    - Simplifying radicals
    - Adding and subtracting radical expressions
    - Multiplying radicals
    - Dividing radicals

2. https://www.khanacademy.org/ Click on "Courses" in the upper left corner. Select Algebra 1. In the menu on the left, choose the topic "Rational exponents and radicals".