

DIY: Vector Basics

This review covers the definition of a vector, graphical and algebraic representations, adding vectors, scalar multiples, dot product, and cross product for two and three dimensional vectors, along with some physics applications. They are followed by several practice problems for you to try, covering all the basic concepts covered in the videos, with answers and detailed solutions. Some additional resources are included for more practice at the end. To review the basics of vectors, watch the following set of YouTube videos. (Note for your security: all hyperlink addresses should begin with <https://youtube.com/watch?v=>)

1. <https://www.youtube.com/watch?v=pimr9I92GZY> (Patrick JMT) Part 1
<https://www.youtube.com/watch?v=odhAVmAAhb4&t=66s> (Patrick JMT) Part 2
<https://www.youtube.com/watch?v=IUzoNG4-MIc&t=23s> (Patrick JMT) Part 3

This series is an introduction to vectors. It includes graphical representation, graphical addition of vectors, scalar multiples, algebraic representation of vectors in component form, finding the component form of a vector between two points, addition and scalar multiplication in component form. Note: the term “**resultant**” which this presenter does not use, means the vector resulting from the addition of two or more vectors.

- 2.) Vector Addition by Parallelogram Method <https://www.youtube.com/watch?v=RqFybu5kVjk> (Organic Chemistry Tutor) (Trigonometric representation of vectors and vector addition)
- 3.) <https://www.youtube.com/watch?v=WxWJorOVlj8&t=28s> (Patrick JMT) Finding magnitude and direction from component form. Comment: In his example, the presenter uses a Quadrant I reference angle to find the Quadrant II angle the vector makes with the positive x-axis.
- 4.) Components <https://www.youtube.com/watch?v=MkFbisCvjFc> (Khan Academy) Finding the component form of a vector from a magnitude and direction.
- 5.) Unit Vectors <https://www.youtube.com/watch?v=iAeKI5h2SJM> (Mario’s Math Tutoring)
- 6.) \hat{i} , \hat{j} form for vectors in component form; addition in component form.
<https://www.youtube.com/watch?v=FaF3v-ezbSk> (Khan Academy) **Note:** presenter uses \hat{i} and \hat{j} . Some other notation forms are \vec{i} and \vec{j} , or \mathbf{i} and \mathbf{j} (boldface). These unit vectors are called “basis vectors”. A set of basis vectors are two (for planar vectors) vectors with which all other planar vectors can be written by adding scalar multiples of the basis vectors. For example, for planar vectors, \mathbf{i} and \mathbf{j} can be basis vectors, since all 2-dimensional vectors can be written using \mathbf{i} and \mathbf{j} . A vector of magnitude 4 making an angle of 30° with the positive x-axis can be written as $4\cos 30^\circ \mathbf{i} + 4\sin 30^\circ \mathbf{j} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$.

7.) three-dimensional vectors, notation, and applications

<https://www.youtube.com/watch?v=vtWzYguOAtw> (Math Fortress) (Part 1)

<https://www.youtube.com/watch?v=zjSE3zyWT9I> (Part 2) problems using 3-dimensional vectors

https://www.youtube.com/watch?v=tbE5KCecz_c (Part 3) more 3-D vector application problems

also suggested: <https://www.youtube.com/watch?v=cQVTMmGurCQ> (Dr. Chris Tisdell)

This presenter uses the column matrix representation for a vector, so $\langle x, y, z \rangle$ becomes $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Since i is a unit

vector in the positive x-direction, it's column matrix representation would be $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. j is a unit vector in the

positive y-direction, so it's column vector representation would be $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and k is the unit vector in the positive z-

direction, so would be $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

8.) Dot Product https://www.youtube.com/watch?v=WNulhXo39_k (Khan Academy)

9.) Finding the angle between two vectors using Dot Product <https://www.youtube.com/watch?v=4hIh8ujylWE>

10.) Cross Product <https://www.youtube.com/watch?v=pWbOisq1MJU>

11.) Properties of Cross Product <https://www.youtube.com/watch?v=WeS4J4-Psgs>

12.) Finding work using vectors - https://www.youtube.com/watch?v=wt_89nyzL_8

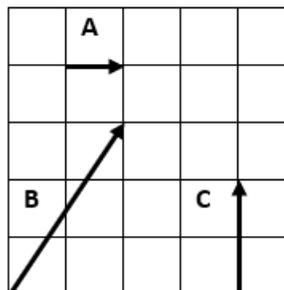
13.) Finding torque using vectors - <https://www.youtube.com/watch?v=2LqtwgrDRQ8>

Practice problems

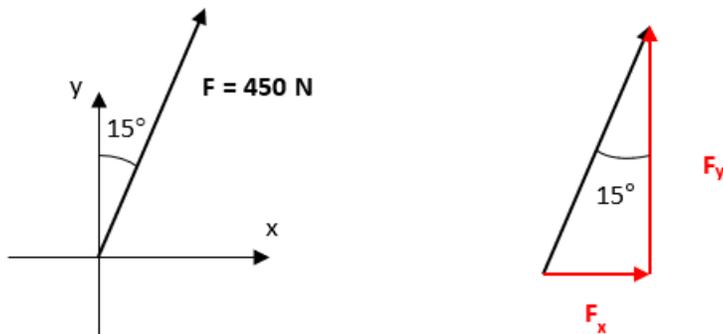
1. Find:

a) $\mathbf{D} = \mathbf{A} + \mathbf{B}$

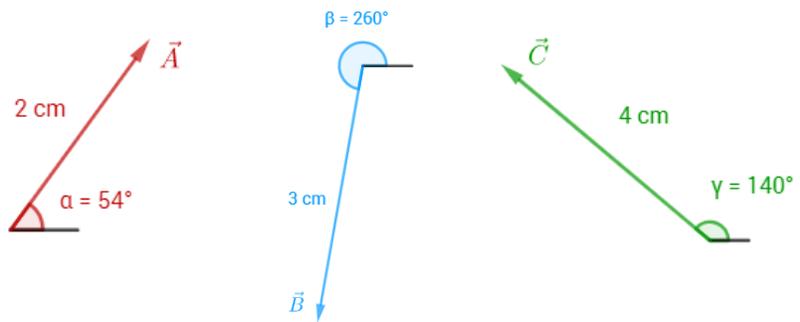
b) $\mathbf{E} = 2\mathbf{C} + 3\mathbf{A} - \mathbf{B}$



2. A weightlifter applies a force of 450 N to a barbell at an angle of 15° to the vertical. What are the x – and y – components of the force?



3. Given the following vectors:



Using the head to tail or parallelogram method find the length and angle from the positive x-axis of the resultant of the following combination of vectors:

- $\mathbf{A + B}$
- $\mathbf{2B + 3C}$
- $\mathbf{C - A}$

4. Solve each of the parts of Question 3 using the component method.

5. Find the dot product of the following pairs of vectors:

- $\mathbf{U = -7i + 6j, V = -10i + j}$
- $\mathbf{U = 4i + 2j, V = 7i}$
- $\mathbf{U = -8i + 5j + -10k, V = 9i + -5j - 9k}$

6. For each of the pairs of vectors in problem 5, compute the angle between them.

7. For each of the following pairs of vectors compute the cross product between them.

a.) $\mathbf{U} = 3\mathbf{i} + -7\mathbf{j} + 10\mathbf{k}$, $\mathbf{V} = 3\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$

b.) $\mathbf{U} = -8\mathbf{i} + 5\mathbf{j} + -10\mathbf{k}$, $\mathbf{V} = 9\mathbf{i} + -5\mathbf{j} - 9\mathbf{k}$

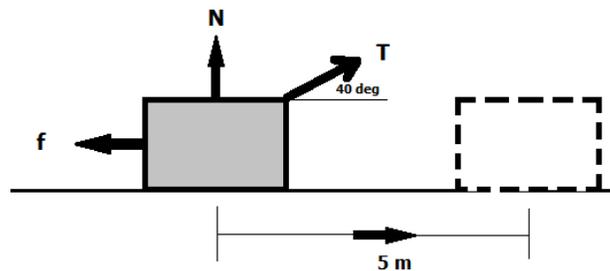
c.) $\mathbf{U} = 3\mathbf{i} + 7\mathbf{j} + -2\mathbf{k}$, $\mathbf{V} = 5\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

8.) Find the area of the parallelograms formed by the following pairs of vectors:

a.) $\mathbf{U} = -5\mathbf{i} + 8\mathbf{j}$, $\mathbf{V} = -9\mathbf{i} - 7\mathbf{j}$

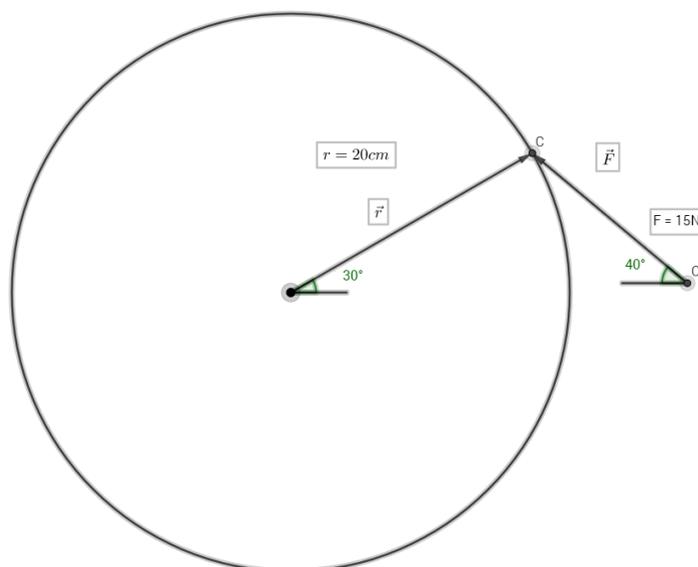
b.) $\mathbf{U} = 3\mathbf{i} + -7\mathbf{j} + 10\mathbf{k}$, $\mathbf{V} = 3\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$

9.) A block is pulled 5m to the right with a force of 150N acting along a rope angled 40° to the horizontal. A normal force of 50 N acts upward, and a friction force of 20N acts in the opposite direction of travel. Compute the work done by each of these forces.



10.) A wrench 25 cm long has a force of 20N applied perpendicular to its end to turn a bolt. Calculate the magnitude of the torque on the bolt.

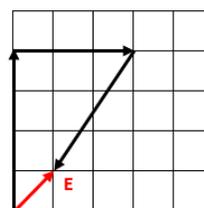
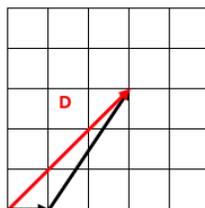
11.) A disk 40 cm in diameter experiences a force on its edge at a point 30° above the positive x-direction. The force has magnitude 15N and is directed 40° above the negative x direction. Compute the magnitude of the torque produced about the disk's center.



12. An object has experiences a force of $83\mathbf{i} - 92\mathbf{j} + 25\mathbf{k}$ N at a location $-.70\mathbf{i} - .38\mathbf{j} - .57\mathbf{k}$ m from its center. Calculate the resultant torque vector and its magnitude.

Answers

1. a. $\mathbf{D} = 3\mathbf{i} + 3\mathbf{j}$ b. $\mathbf{E} = \mathbf{i} + \mathbf{j}$



2. $F_x \approx 116.5$ N; $F_y \approx 434.7$ N

3. a. magnitude = 1.488 direction: $\theta = 296.1^\circ$
 b. magnitude = 10.39 direction: $\theta = 170^\circ$
 c. magnitude = 4.35 direction: $\theta = 167.33^\circ$

4. (same answers as Q 3.

5. a. 76 b. 28 c. -7

6. a. 34.89° b. 26.57° c. 92.13°
7. a. $-20i + 6k$ b. $-95i - 162j - 5k$ c. $-26i - 4j - 53k$
8. a. 107 b. 20.88
9. a. $W_T = 574.53 \text{ J}$. b. $W_N = 0 \text{ J}$. c. $W_f = -100 \text{ J}$.
10. 5 Nm
11. 2.82 Nm
12. a. $\tau = -61.94i - 29.81j + 95.94k$ b. $\|\tau\| = 118 \text{ Nm}$

(Detailed solutions to these problems begin on the next page.)

Detailed Solutions:

1. a.

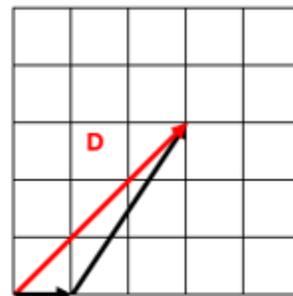
We can add the vectors graphically using the head-to-tail method. From the graph we see that

$$\underline{D_x} = 3 \text{ and } \underline{D_y} = 3 \quad \text{or} \quad \mathbf{D} = 3\mathbf{i} + 3\mathbf{j}$$

Without using a graph, we can add the x – and y – components of the vectors separately.

$$\begin{array}{l} \underline{D_x} = A_x + B_x \\ \underline{D_x} = 1 + 2 \end{array} \qquad \begin{array}{l} \underline{D_y} = A_y + B_y \\ \underline{D_y} = 0 + 3 \end{array}$$

$$\underline{D_x} = 3 \text{ and } \underline{D_y} = 3 \quad \text{or} \quad \mathbf{D} = 3\mathbf{i} + 3\mathbf{j}$$



b.

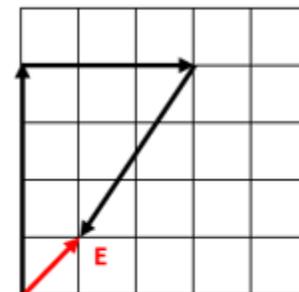
From the graph we see that

$$\underline{E_x} = 1 \text{ and } \underline{E_y} = 1 \quad \text{or} \quad \mathbf{E} = \mathbf{i} + \mathbf{j}$$

Alternatively

$$\begin{array}{l} \underline{E_x} = 2C_x + 3A_x - B_x \\ \underline{E_x} = 2(0) + 3(1) - (2) \end{array} \qquad \begin{array}{l} \underline{E_y} = 2C_y + 3A_y - B_y \\ \underline{E_y} = 2(2) + 3(0) - (3) \end{array}$$

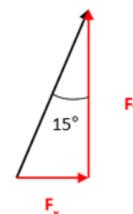
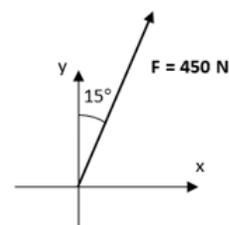
$$\underline{E_x} = 1 \text{ and } \underline{E_y} = 1 \quad \text{or} \quad \mathbf{E} = \mathbf{i} + \mathbf{j}$$



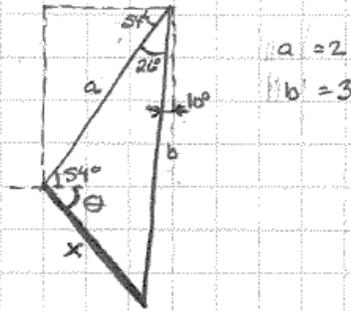
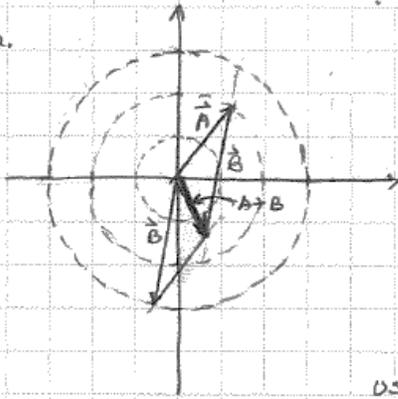
2.

Consider the 450 N force as the sum of its x – and y – components. We can use trigonometric functions to find the components.

$$\begin{array}{l} \sin 15^\circ = \frac{F_x}{F} \\ F_x = (450 \text{ N}) \sin 15^\circ \\ F_x \approx 116.5 \text{ N} \end{array} \quad \text{and} \quad \begin{array}{l} \cos 15^\circ = \frac{F_y}{F} \\ F_y = (450 \text{ N}) \cos 15^\circ \\ F_y \approx 434.7 \text{ N} \end{array}$$



3. a.



using the law of cosines:

$$\begin{aligned} x^2 &= a^2 + b^2 - 2ab \cos 26^\circ \\ &= 2^2 + 3^2 - 2(2)(3)(.898794) \\ &= 4 + 9 - 12(.898794) \\ &= 13 - 10.7855 \\ &= 2.2145 \end{aligned}$$

$$x = \|\vec{A} + \vec{B}\| = 1.48811 \approx \boxed{1.5 \text{ cm}}$$

To find the direction of $\vec{A} + \vec{B}$, use the law of sines:

$$\frac{x}{\sin 26^\circ} = \frac{b}{\sin(54^\circ + \theta)}$$

$$\frac{1.48811}{\sin 26^\circ} = \frac{3}{\sin(54^\circ + \theta)} \Rightarrow \sin(54^\circ + \theta) = \frac{3(.43837)}{1.48811} = .8837$$

$$54^\circ + \theta = 62.1^\circ \text{ or } (180^\circ - 62.1^\circ)^*$$

$$= 62.1^\circ \text{ or } 117.9^\circ$$

$$\theta = 8.1^\circ \text{ or } 63.9^\circ$$

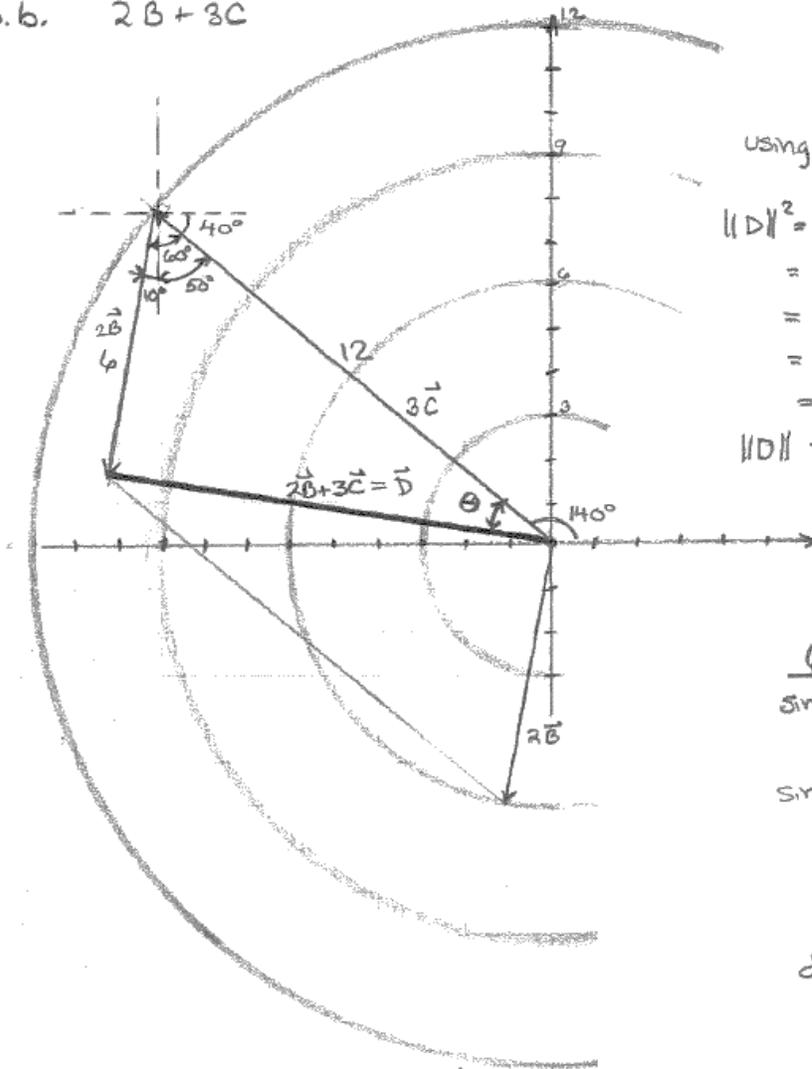
* $(54^\circ + \theta)$ could be a ϕ .I angle or a ϕ .II angle since sine is + in both.

from the sketch, it

looks like $\theta = 63.9^\circ \approx 64^\circ$

$$\begin{aligned} \text{direction of } \vec{A} + \vec{B} &= 360^\circ - 64^\circ \\ &= \boxed{296^\circ} \end{aligned}$$

3.6. $2\vec{B} + 3\vec{C}$



using the law of cosines,

$$\begin{aligned} \|\vec{D}\|^2 &= \|\vec{3C}\|^2 + \|\vec{2B}\|^2 - 2\|\vec{2B}\|\|\vec{3C}\|\cos 60^\circ \\ &= 12^2 + 6^2 - 2(6)(12)\cos 60^\circ \\ &= 144 + 36 - 144\left(\frac{1}{2}\right) \\ &= 180 - 72 \\ &= 108 \\ \|\vec{D}\| &= \sqrt{108} = 3\sqrt{12} = 6\sqrt{3} \\ \|\vec{D}\| &\approx \boxed{10.4} \end{aligned}$$

Direction:

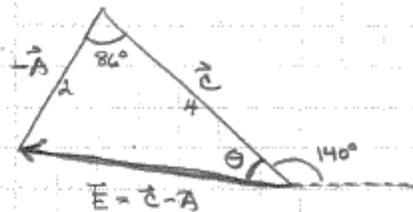
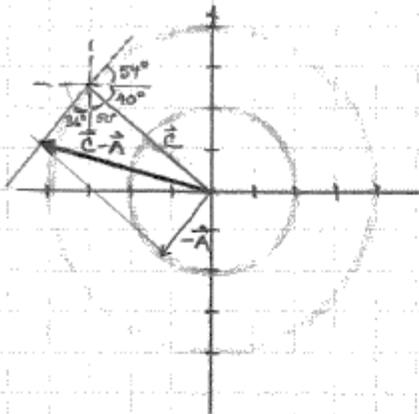
$$\frac{6}{\sin \theta} = \frac{10.4}{\sin 60^\circ}$$

$$\sin \theta = \frac{6 \sin 60^\circ}{6\sqrt{3}} = \frac{6(\sqrt{3}/2)}{6\sqrt{3}}$$

$$\approx \frac{1}{2} \quad \theta = 30^\circ$$

$$\begin{aligned} \text{direction of } \vec{D} &= 140^\circ + 30^\circ \\ &= \boxed{170^\circ} \end{aligned}$$

3.c. $\vec{c} - \vec{a}$



$$\begin{aligned} \|\vec{E}\|^2 &= 2^2 + 4^2 - 2(2)(4)\cos 86^\circ \\ &= 4 + 16 - 16(0.06976) \\ &= 18.884 \\ \|\vec{E}\| &= 4.34556 \approx \boxed{4.3} \text{ cm} \end{aligned}$$

Direction: $\frac{2}{\sin \theta} = \frac{4.34556}{\sin 86^\circ} \rightarrow \sin \theta = \frac{2 \sin 86^\circ}{4.34556}$
 $= .4591$
 $\theta = 27.33^\circ$
 direction of $\vec{E} = 140^\circ + 27.33^\circ$
 $\approx \boxed{167^\circ}$

4. a. using components. $\vec{A} = \langle 2 \cos 54^\circ, 2 \sin 54^\circ \rangle = \langle 1.17557, 1.618034 \rangle$

$\vec{B} = \langle 3 \cos 260^\circ, 3 \sin 260^\circ \rangle = \langle -0.520945, -2.954423 \rangle$

$\vec{C} = \langle 4 \cos 140^\circ, 4 \sin 140^\circ \rangle = \langle -3.064178, 2.57115 \rangle$

$\vec{A} + \vec{B} = \langle 1.17557 - 0.52095, 1.618034 - 2.954423 \rangle$
 $= \langle 0.65462, -1.33639 \rangle$

$\|\vec{A} + \vec{B}\| = \sqrt{(0.65462)^2 + (-1.33639)^2} = \boxed{1.488} \approx 1.5$

direction: $\tan \theta = \frac{y}{x} = \frac{-1.33639}{0.65462} = -2.04147$ $\theta = \frac{360^\circ - 63.9^\circ}{\text{sm } \theta \cdot V} = \boxed{296.1^\circ}$

$$4.b. \quad 2\vec{B} + 3\vec{C} = \langle 2(-.520945) + 3(-3.064178), 2(-2.954423) + 3(2.57115) \rangle$$

$$= \langle -10.234424, 1.804604 \rangle$$

$$\|2\vec{B} + 3\vec{C}\| = \sqrt{(-10.2344)^2 + (1.8046)^2} = \boxed{10.39}$$

direction: $\tan \theta = \frac{1.804604}{-10.2344}$ (θ in $Q.II$)

$$\tan \theta = -0.17633$$

$$\theta = 180^\circ - 10^\circ = \boxed{170^\circ}$$

$$c. \quad \vec{C} - \vec{A} = \langle -3.064178 - 1.17557, 2.57115 - 1.618034 \rangle$$

$$= \langle -4.239748, 0.953116 \rangle$$

$$\|\vec{C} - \vec{A}\| = \sqrt{(-4.23975)^2 + (.95312)^2} = 4.34556 \approx \boxed{4.35}$$

direction: $\tan \theta = \frac{y}{x} = \frac{.95312}{-4.23975} = -.2248$ (θ in $Q.II$)

$$\theta = 180^\circ - \arctan .2248$$

$$= 180^\circ - 12.67^\circ \approx \boxed{167.3^\circ}$$

5.) compute $U \cdot V$ for the following pairs of vectors.

a.) $U = -7i + 6j, V = -10i + j.$

To compute the dot product we multiply the components and add them.

$$U \cdot V = (-7)(-10) + (6)(1) = 70 + 6 = 76$$

b.) $U = 4i + 2j, V = 7i$

Note there is no y-component for V , so it is implied to be zero.

$$U \cdot V = (4)(7) + (2)(0) = 28 + 0 = 28$$

c.) $\mathbf{U} = -8\mathbf{i} + 5\mathbf{j} + -10\mathbf{k}, \mathbf{V} = 9\mathbf{i} + -5\mathbf{j} - 9\mathbf{k}$

It works the same way in higher dimensions.

$$\mathbf{U} \cdot \mathbf{V} = (-8)(9) + (5)(-5) + (-10)(-9) = -72 - 25 + 90 = -7$$

6.) For each of the pairs of vectors in problem 5 compute the angle between them.

To find the angle between two vectors we use the identity

$$\mathbf{U} \cdot \mathbf{V} = \|\mathbf{U}\| \|\mathbf{V}\| \cos\theta$$

We solve this for the angle θ .

$$\theta = \arccos \frac{\mathbf{U} \cdot \mathbf{V}}{\|\mathbf{U}\| \|\mathbf{V}\|}$$

a.) $\mathbf{U} = -7\mathbf{i} + 6\mathbf{j}, \mathbf{V} = -10\mathbf{i} + \mathbf{j}$.

From problem 3: $\mathbf{U} \cdot \mathbf{V} = 76$

$$\|\mathbf{U}\| = \sqrt{(-7)^2 + (6)^2} = \sqrt{49 + 36} = \sqrt{85}$$

$$\|\mathbf{V}\| = \sqrt{(-10)^2 + (1)^2} = \sqrt{100 + 1} = \sqrt{101}$$

Therefore

$$\theta = \arccos \frac{76}{\sqrt{85}\sqrt{101}} \approx 34.89^\circ$$

b.) $\mathbf{U} = 4\mathbf{i} + 2\mathbf{j}, \mathbf{V} = 7\mathbf{i}$

From problem 3: $\mathbf{U} \cdot \mathbf{V} = 28$

$$\|\mathbf{U}\| = \sqrt{(4)^2 + (2)^2} = \sqrt{16 + 4} = \sqrt{20}$$

$$\|\mathbf{V}\| = \sqrt{(7)^2 + (0)^2} = \sqrt{49} = 7$$

Therefore

$$\theta = \arccos \frac{28}{7\sqrt{20}} \approx 26.57^\circ$$

c.) $U = -8i + 5j + -10k, V = 9i + -5j - 9k$

From problem 5: $U \cdot V = -7$

$$||U|| = \sqrt{(-8)^2 + (5)^2 + (-10)^2} = \sqrt{64 + 25 + 100} = \sqrt{189}$$

$$||V|| = \sqrt{(9)^2 + (-5)^2 + (-9)^2} = \sqrt{81 + 25 + 81} = \sqrt{187}$$

Therefore

$$\theta = \arccos \frac{-7}{\sqrt{189}\sqrt{187}} \approx 92.13^\circ$$

7. Compute $U \times V$ for each of the following pairs of vectors.

a.) $U = 3i + -7j + 10k, V = 3i - 5j + 10k$

To compute the cross product we compute the determinant of the 3×3 matrix

$$\begin{vmatrix} i & j & k \\ 3 & -7 & 10 \\ 3 & -5 & 10 \end{vmatrix}$$

Using the cofactor method.

$$\begin{aligned} U \times V &= i \begin{vmatrix} -7 & 10 \\ -5 & 10 \end{vmatrix} - j \begin{vmatrix} 3 & 10 \\ 3 & 10 \end{vmatrix} + k \begin{vmatrix} 3 & -7 \\ 3 & -5 \end{vmatrix} \\ &= -20i + 0j + 6k \end{aligned}$$

b.) $U = -8i + 5j + -10k, V = 9i - 5j - 9k$

$$U \times V = \begin{vmatrix} i & j & k \\ -8 & 5 & -10 \\ 9 & -5 & -9 \end{vmatrix}$$

$$\begin{aligned} i \begin{vmatrix} 5 & -10 \\ -5 & -9 \end{vmatrix} - j \begin{vmatrix} -8 & -10 \\ 9 & -9 \end{vmatrix} + k \begin{vmatrix} -8 & 5 \\ 9 & -5 \end{vmatrix} \\ = -95i - 162j - 5k \end{aligned}$$

c.) $U = 3i + 7j - 2k, V = 5i - 6j - 2k$

$$\begin{aligned} U \times V &= \begin{vmatrix} i & j & k \\ 3 & 7 & -2 \\ 5 & -6 & -2 \end{vmatrix} \\ &= i \begin{vmatrix} 7 & -2 \\ -6 & -2 \end{vmatrix} - j \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix} + k \begin{vmatrix} 3 & 7 \\ 5 & -6 \end{vmatrix} \\ &= -26i - 4j - 53k \end{aligned}$$

8.) Find the area of the parallelograms formed by the following pairs of vectors:

a.) $U = -5i + 8j, V = -9i - 7j$

The area of the parallelogram formed by two vectors is given by the magnitude of the cross product.

$$\begin{aligned} U \times V &= \begin{vmatrix} i & j & k \\ -5 & 8 & 0 \\ -9 & -7 & 0 \end{vmatrix} \\ &= i \begin{vmatrix} 8 & 0 \\ -7 & 0 \end{vmatrix} - j \begin{vmatrix} -5 & 0 \\ -9 & 0 \end{vmatrix} + k \begin{vmatrix} -5 & 8 \\ -9 & -7 \end{vmatrix} \\ &= 0i + 0j + 107k \\ \text{Area} &= ||U \times V|| = \sqrt{0^2 + 0^2 + 107^2} = 107 \end{aligned}$$

b.) $U = 3i + -7j + 10k, V = 3i - 5j + 10k$

These are the same vectors from problem 7a.) so their cross product has already been solved as

$$\begin{aligned} U \times V &= -20i + 0j + 6k \\ \text{Area} &= ||U \times V|| = \sqrt{(-20)^2 + (0)^2 + (6)^2} = \sqrt{436} \approx 20.88 \end{aligned}$$

9.) A block is pulled 5m to the right with a force of 150N acting along a rope angled 40° to the horizontal. A normal force of 50 N acts upward, and a friction force of 20N acts in the opposite direction of travel. Compute the work done by each of these forces.

Work is the product of the distance an object moves and the magnitude of the force in the direction of the motion. For a constant force the work can be computed as the dot product of the force with the displacement.

$$W = \mathbf{F} \cdot \mathbf{x} = |\mathbf{F}||\mathbf{x}|\cos\theta$$

(note: $|\mathbf{F}|\cos\theta$ is the magnitude of the component of the force in the direction of motion.)

For the tension force, the angle between the force and direction of travel is 40° .

$$W_T = (150N)(5m) \cos(40^\circ) \approx 574.53J$$

For the normal force, the force act perpendicular to the direction of travel. So the angle is 90°

$$W_N = (50N)(5m) \cos(90^\circ) = 0J$$

And for the friction force, it is in the opposite direction as the displacement direction. The angle is 180° .

$$W_f = (20N)(5m) \cos(180^\circ) = -100J$$

10.) A wrench 25 cm long has a force of 20N applied perpendicular to its end to turn a bolt. Calculate the magnitude of the torque on the bolt.

Torque is the product of \mathbf{r} , the vector from the center of rotation to the point where the force is applied, and the magnitude of the component of the force perpendicular to \mathbf{r} . The torque formula is

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \text{ which is } |\mathbf{r}||\mathbf{F}|\sin\theta$$

(Note that $|\mathbf{F}|\sin\theta$ is the magnitude of the component of the force perpendicular to the line from the force to the center of rotation.)

We will take $\mathbf{r} = .25m \mathbf{i}$ and $\mathbf{F} = 20N \mathbf{j}$.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ .25 & 0 & 0 \\ 0 & 20 & 0 \end{vmatrix} = 5 N \cdot m \mathbf{k}$$

The magnitude of the torque is 5 Nm.

11.) A disk 20 cm in diameter experiences a force on its edge at a point 30° above the positive x-direction. The force has magnitude 15N and is directed 40° above the negative x direction. Compute the magnitude of the torque produced about the disk's center.

To use the torque formula

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

We need the vectors in their component forms.

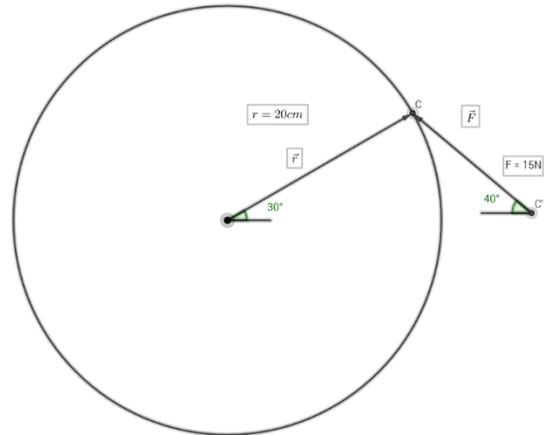
$$\mathbf{r} = (.20 \cos(30^\circ) \mathbf{i} + .20 \sin(30^\circ) \mathbf{j})\text{m}$$

$$\mathbf{F} = (15 \cos(140^\circ) \mathbf{i} + 15 \sin(140^\circ) \mathbf{j})\text{N}$$

Note 140° is used instead of 40° as that is the angle measure off of the positive x-axis.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ .173 & .100 & 0 \\ -11.5 & 9.64 & 0 \end{vmatrix} = 2.82 \text{ N} \cdot \text{m} \mathbf{k}$$

The magnitude of the torque is 2.82 Nm.



12. An object has experiences a force of $83\mathbf{i} - 92\mathbf{j} + 25\mathbf{k}$ N at a location $-.70\mathbf{i} - .38\mathbf{j} - .57\mathbf{k}$ m from its center. Calculate the resultant torque vector and its magnitude.

We have

$$\mathbf{r} = -.70\mathbf{i} - .38\mathbf{j} - .57\mathbf{k}$$

and

$$\mathbf{F} = 83\mathbf{i} - 92\mathbf{j} + 25\mathbf{k}$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = \mathbf{U} \times \mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -.70 & -.38 & -.57 \\ 83 & -92 & 25 \end{vmatrix} = -43\mathbf{i} - 30\mathbf{j} + 33\mathbf{k}$$

And its magnitude is given as

$$|\boldsymbol{\tau}| = \sqrt{(-61.94)^2 + (-29.81)^2 + (95.94)^2} = 118 \text{ Nm}$$

Additional Resources for Practice:

1. <https://www.kutasoftware.com/freeipc.html>

See the sections for “Vectors” and “Three-Dimensional Vectors

2. <http://dev.physicslab.org/PracticeProblems/Worksheets/APC/Vectors/crossproduct.aspx>

On finding torque